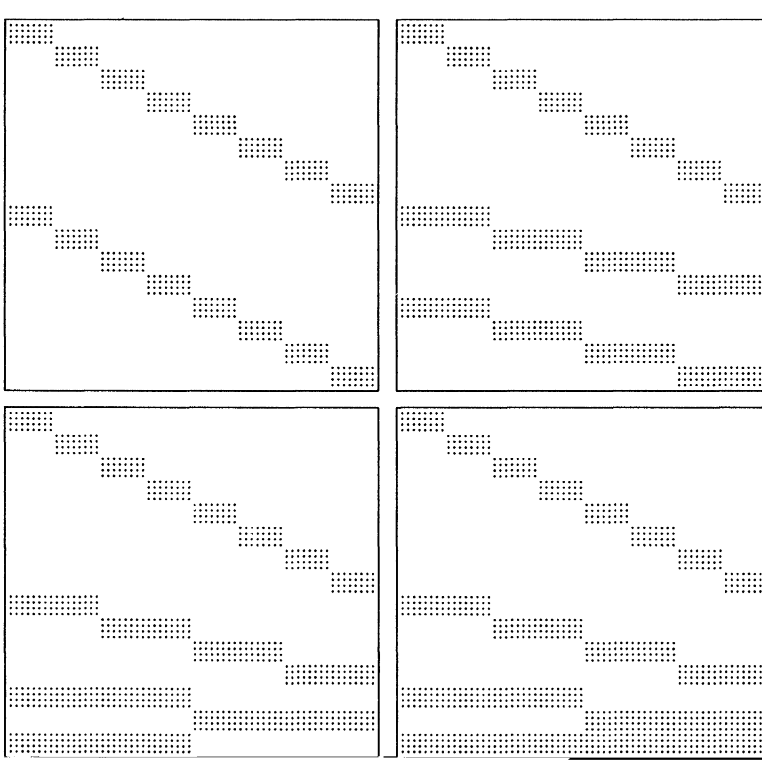
more moments to vanish. Namely, out of the k vectors nonzero on $x_1, \ldots, x_{2k}$, we require that one have k vanishing moments, a second have $k+1$, a third have $k+2$, and so forth, and the kth have $2k-1$ vanishing moments. We place the same condition on the k basis vectors nonzero on $x_{2k+1}, \ldots, x_{4k}$, and so on, for each block of k basis vectors among the $n-k$ basis vectors with zero moments.

We construct the basis by construction of a finite sequence of bases (shown in Fig. 2), each obtained by a number of orthogonalizations. The first basis results from $n/(2k)$ Gram–Schmidt orthogonalizations of $2k$ vectors each. In particular, the vectors $\langle x_1{}^j, \ldots, x_{2k}{}^j \rangle$ for $j = 0, \ldots, 2k-1$ are orthogonalized, the vectors

$\langle x_{2k+1}{}^j, \ldots, x_{4k}{}^j \rangle$ for $j = 0, \ldots, 2k-1$ are orthogonalized, and so forth, up to the vectors $\langle x_{n-2k+1}{}^j, \ldots, x_n{}^j \rangle$ for $j = 0, \ldots, 2k-1$, which are orthogonalized.



where I_m is the $m \times m$ identity matrix and the $n/2 \times n/2$ matrix U_2' is given by the formula

[illegible]

where $n_2 = n/(4k)$, ${U_{2,i}}^T = \text{Orth}\,(M_{2,i})$, and the $2k \times 2k$ matrix $M_{2,i}$ is given by

$$M_{2,i} = \begin{pmatrix} {U_{1,2i-1}}^U M_{1,2i-1} \\ {U_{1,2i}}^U M_{1,2i} \end{pmatrix}$$

for $i = 1, \ldots, n/(4k)$. In general, the ith basis matrix, for $i = 2, \ldots, \log_2(n/k)$, is [illegible]

[illegible]

$U_i = U_i'$ with U_i' defined by the formula

where $\mu_{j,i} = (x_{1+(i-1)k2^j} + x_{ik2^j})/2, \sigma_{j,i} = (x_{ik2^j} - x_{1+(i-1)k2^j})/2$, and the matrix $M_{j,i}$ is defined by (1) and (3) in §1. The matrix $U_{j,i}$ is given by the formula

$$U_{j,i}{}^T = \text{Orth}(M'_{j,i}), \tag{21}$$

which is equivalent to the definition given by (2). This equivalence immediately follows [illegible]

[illegible]

The matrices $M'_{1,i}$ for $i = 1, \ldots, n/(2k)$ are actually computed by the formula

$$\begin{pmatrix} 1 & \frac{x_{s_i+1}-\mu_{1,i}}{\sigma_{1,i}} & \cdots & \left(\frac{x_{s_i+1}-\mu_{1,i}}{\sigma_{1,i}}\right)^{2k-1} \\ \end{pmatrix}$$

[illegible]

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