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Keywords:

1. I t ct

( ): 2 2n

n $n > 1$,

z, $z_n \to z$, U z

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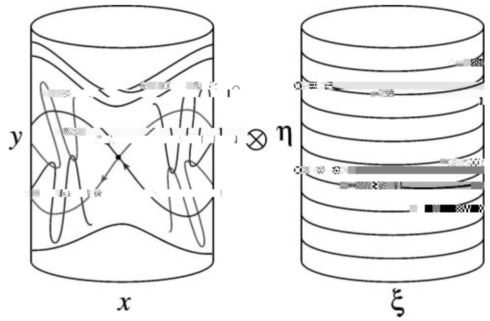
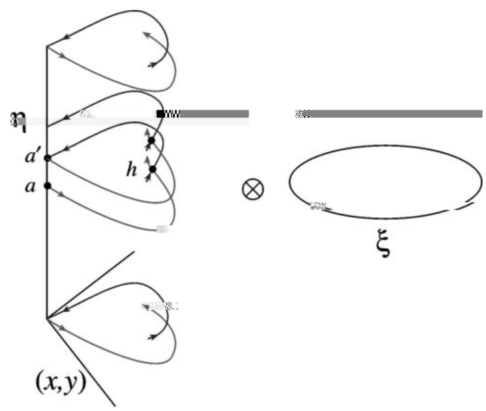


Fig. 1. … (1) … $h = 0$ … (x, y), … (ξ, η).

… $T^2 \times R^2$ …

$$x' = x + y', \qquad \xi' = \xi + \eta', \qquad y' = y - k(1 + h \ldots \xi) \ldots x, \qquad \eta' = \eta - kh($$



. 2. (1) $h \neq 0$. ξ, C_a. C_a $C_{a'}$ h.

a' a.

$(x, y) \mapsto (x', y')$,

$$x' = x + \frac{1}{\varepsilon} y', \qquad y' = y + V(x). \tag{4}$$

$n = 1$, $V(x) = k(x)$, [13].

3. A t- t ab t

A 3,4,15 (3)

= ∗=1,91.4887 -291.4887 - 220.1(87 -)-17,1.4887 - 1.4887 - 87 - 1 16 0 0 7.5716 177.3568 5 5256 6640

4. C[illegible]

[illegible]

that C is not identically zero. Then given any $a < b$, there is a nonzero measure of initial states (ξ_0, ξ_1) and a sequence $c_t \in .(V)_+ \cup .(V)_-$ such that the solution of (14) *has momenta, $\eta_t = T_2(\xi_{t-1}, \xi_t)$ satisfying $\eta_0 < a$ and $\eta_T > b$ for some time T.*

P [illegible] $c_- \in .(V)_-$ [illegible] $c_+ \in .(V)_+$ [illegible] ξ_t, [illegible] $x_t = c_\pm$ [illegible] $(C(\xi_t)) = \pm 1$. [illegible] (14) [illegible]

$$\tilde{L}(\xi, \xi') = T(\xi, \xi') + W(\xi) + \tilde{C}(\xi),$$

[illegible] $\tilde{C}$ [illegible]

$$\tilde{C} = V(c_\pm(\xi)) \quad C(\xi) \geq 0.$$

[illegible] $\tilde{C}(\xi + 2\pi) - \tilde{C}(\xi) > 0$, [illegible] $= \tilde{L}(\xi + 2\pi, \xi' + 2\pi) - \tilde{L}(\xi, \xi')$ [illegible] 5, [illegible] □

4.2. Standard example

[illegible]

$$L(x, x', \xi, \xi') = \tfrac{1}{2}\varepsilon(x' - x)^2 + \tfrac{1}{2}(\xi' - \xi)^2 + k \quad x(1 + h \quad \xi), \tag{15}$$

[illegible] $k > 0$ [illegible] $h > 0$ [illegible] (1), [illegible] (15) [illegible] h

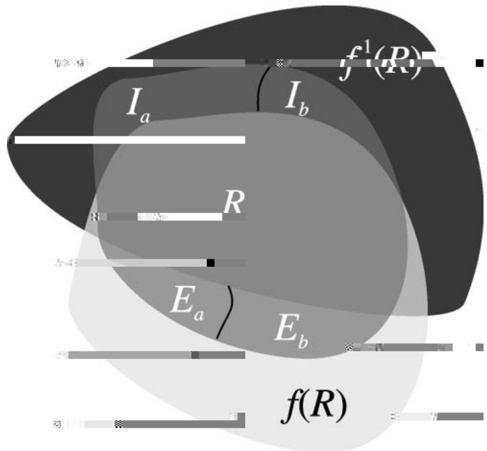
$\xi_t \to \pi$, $\eta^* = 2\pi m$. $(0, 2\pi m)$ $(\pi, 2\pi m)$, (16). $\eta^* = (2m+1)\pi$, $(0, \eta^*) \mapsto (\pi, \eta^*)$. (16) ( ). (16)

(16), g^* g g^* g. (16) 2π $\mathrm{T}^2 = \{(\xi, \eta) : 0 < \xi, \eta < 2\pi\}$.

$$\eta^* = (\eta_{t+1} - \eta_t)^* = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} (U'(\xi_t)).$$

$$\langle \eta^* \rangle = \frac{1}{4\pi^2} \int_{\mathrm{T}^2} U'(\xi) = \frac{}{2\pi}.$$

(16) .012



4. ... R.

$$E = \{z \in R : f(z) \notin R\} = R \setminus f^{-1}(R).$$

... f ... μ, ... R ...:

$$\mu(E)=\mu(R \setminus f^{-1}(R))=\mu(R) - \mu(R \cap f^{-1}(R)) = \mu(R) - \mu(f(R) \cap R) = \mu(R \setminus f(R)) = \mu(I). \quad (17)$$

... S^t ... t ... I ... *stays* ... R ... t ...

$$S^0 = I, \qquad S^t = f(S^{t-1}) \cap R = f(S^{t-1} \setminus E).$$

... S^t ... $f^{-j}(S^t)$ ... R ... $j = 0, \ldots, t$, ... $(t+1)^{st}$, ... S^t ... R, ... $\sum_{t=0}^{\infty} \mu(S^t) < \mu(R)$. ... $\mu(S^t) \to 0$ ... $t \to \infty$. ... I ... R ... E.

E_b

$$\mu(p(I_a) \cap E_b) = \mu(p(I_a)) - \mu(p(I_a) \cap E_a) \geq \mu(I_a) - \mu(E_a). \qquad \square$$

A f_t R, t

$$I_t = R \setminus f_t(R), \qquad E_t = R \setminus f_t^{-1}(R).$$

f_t (17), R k t S_k^t.

$$S_k^k = I_{k-1}, \qquad S_k^{t+1} = f_t(S_k^t \setminus E_t).$$

A S_k^t t $k \leq t$ R

$$\sum_{k=-\infty}^{t} \mu(S_k^t) < \mu(R) \tag{18}$$

t.

Lemma 4. *Let f_t be a sequence of measure-preserving homeomorphisms, and R a measurable set with incoming sets I_t and exit sets* t 1012(

5.2. Maps of the cylinder

… C, … C … $f(C)$

$$= {}_C\; y', x' - y, x.$$

… C.

… C … T … B. … $U \subset T$ …

$$U = \{z \in T : f^{-1}(z) \in B\}.$$

… $D \subset B$ …:

$$D = \{z \in B : f^{-1}(z) \in T\}.$$

… $\mu(U) - \mu(D) =$ …

… 4 …

Corollary 5. *Suppose that f_t is a sequence of area and end-preserving homeomorphisms of the cylinder, and that the net flux $_t \geq \delta > 0$. Let A denote the annulus bounded by the circles $\{y = a\}$ and $\{y = b\}$ where $a < b$. Then, there is a set of positive measure of orbits that cross the annulus.*

Proof. … $U_t(a)$ … $D_t(a)$ … $\{y = a\}$ … f_t, … b, … 5. … A … $I_t = U_t(a) \cup D_t(b)$, … $E_t = f_t^{-1}(D_t(a)) \cup f_t^{-1}(U_t(b))$. … $\mu(U_t(b)) \geq \delta > 0$, … 4, … a … b … $f_t^{-1}(U_t(b))$. □

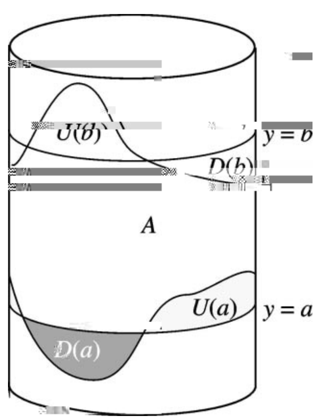
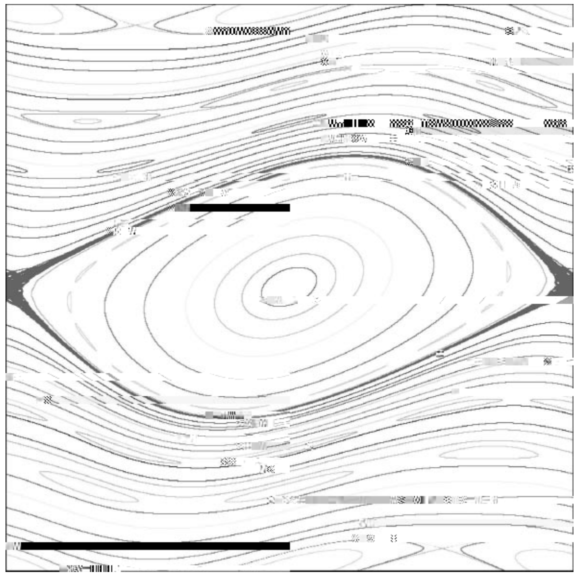


Fig. 5. …



. 6. $k = 0.5$.

5.3. Standard map with net flux

(4) $(0, 2\pi) \times \mathrm{R}$. $= V(2\pi) - V(0)$. V :

$$x' = x + y', \qquad y' = y - k\ (x) + \frac{1}{2\pi}\ .$$

$= 0$, $k < k_{cr} \approx 0.971635406$ $k = 0.5$. 6. 5, $\neq 0$ $y = 0$ $y = 2\pi$. m, $f(x, y + 2\pi m) = f(x, y) + 2\pi(m, m)$. 7, $y = 0$

$$x = \qquad \overline{2\pi k}$$

$= 2\pi k$.

6. P t c t

(12) ε. $z_{t-1} = (x_{t-1}, x_t, \xi_{t-1}, \xi_t)$

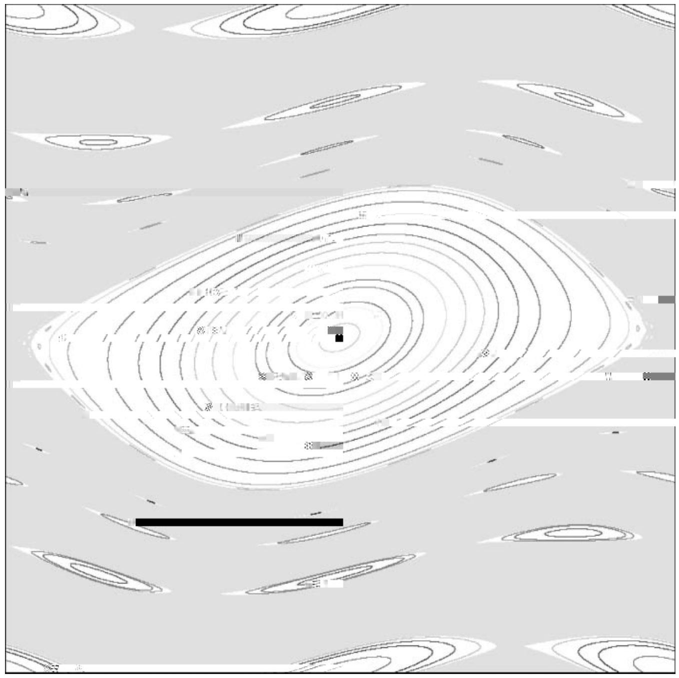


Fig. 7. ... $k = 0.5$, ... $\mathcal{F} = 4\pi^2/1000$. ... (...).

$$z_t = \phi_\varepsilon(z_{t-1}) = \begin{pmatrix} x_t \\ -x_{t-1} + 2x_t + \dfrac{1}{\varepsilon} V(x_t)/[1 + C(\xi_t)] \\ \xi_t \\ -\xi_{t-1} + 2\xi_t + W(\xi_t) + V(x_t) C(\xi_t) \end{pmatrix}. \tag{19}$$

... $\varepsilon = 0$, ... $x_t = c_t \in$ ...(V) ... ϕ_0. ... $\varepsilon > 0$, ... x- ...

L... a 6. *Suppose that ϕ_ε, given by* (19), *is a C^2 map of T^4, such that $1 + C(\xi) \geq \tau > 0$. Then, for any sequence $\{c_0, c_1, \dots\}$ with $c_t \in$ ...$(V) \cap \mathcal{A}$, any initial condition (ξ_0, ξ_1), and any $\delta > 0$, there exists an orbit $z_t = (x_t, x_{t+1}, \xi_t, \xi_{t+1})$, $t \geq 0$ of ϕ_ε such that*

$$|x_t - c_t| \leq \delta \quad \text{...} \quad t \geq 0,$$

provided

$$0 \leq \varepsilon < \varepsilon_0 = \frac{\sigma\tau}{(4\delta + a)}, \tag{20}$$

where $\sigma(\delta, b) \equiv$ ...$_{t\geq 0}|V(c_t \pm \delta)|$.

a … $1 + C(\xi) \geq \tau$ …
$(1/\varepsilon)\ V(c_{t+1}^{+})\tau > (4\delta + a)$ … (ε) … $\sigma\tau/\varepsilon > 4\delta + a$ … (20).
… W_t … B_{t+1} … W_{t+1}.
… S … W_0 … $S = \mathrm{R}^2 \times (\xi_0, \xi_1) \cap W_0$. …
… z_0 … S … T … S …
… U_t … W_t … $t \geq 1$. … B …
… S …
… W_0, … T … B …

… T … B … W_t …
… S … T … B.
… S … W_t … □

… 2, … (ξ_0, ξ_1) … $\{c_t\} \in$ … (V) …
… $Z_t = (c_t, c_{t+1}, \xi_t, \xi_{t+1})$ …
… ε … $\{Z_t\}$ … $\{z_t\}$ … ϕ …
… T … ε …

T… 7. *Suppose that ϕ_ε satisfies the hypotheses of Lemma 6. Let $Z_t = (c_t, c_{t+1}, \xi_t, \xi_{t+1})$ be an orbit of ϕ_0 with $c_t \in$ … (V). Then for any $T \geq 0$ and $\gamma > 0$, there is a $\delta > 0$ such that for all $\varepsilon < \varepsilon_0(\delta)$ in* (20.5716 0 0 7.5716 439. 1

$|\lambda_t^i| \le r^t$, $r > 1$ $r^2 - wr - 1 = 0$, $w =$ $_x(2 + |$ $W(x)|)$.

$|\lambda_t| \le \frac{1}{2} M\delta^2 r^{2t}$.

$W = 0$, λ_t t^2, T γ δ $|\lambda_t| \le \gamma$ $t \le T$. □

R **a** ξ_t $C(\xi)$,

6.1. Standard example, continued

7 (15), $V(x) = k$ x, $\delta < \pi/2$, $\sigma = k$ δ. $C(\xi) = h$ ξ, $\tau = 1 - h$, $h < 1$. $c_t \in \{0, \pi\}$, (x, y) $a = 2\pi$. 6

$$\varepsilon \le \varepsilon_0 = \frac{k(1-h)\ \ \delta}{4\delta + 2\pi}.$$

7 $M = kh$ $W = 1$

$h < 1$ $\varepsilon \leq \varepsilon_0$. ε 9.

7. C[illegible]

[illegible] [7] [illegible] [6,9,20] [illegible] [19] [illegible]

[illegible] [16] [illegible] (x, y) [illegible] [17].

Ac[illegible]t

[illegible] 1999. [illegible] 4. [illegible] -9971760. [illegible] -9810751.

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[1] [illegible] 5 (1964) 581 585.
[2] [illegible], 1978.
[3] [illegible] (, .), [illegible] 44 [illegible], 1992, 7 54.
[4] [illegible] 86 (1995) 284 296.
[5] [illegible] 12 (2) (1999) 365 388.
[6] [illegible] 14 (1984) 289 304.
[7] [illegible] 209 (2000) 353 392.
[8] [illegible] 4 (1991) 583 590.
[9] [illegible] 38 [illegible], 1992.

[10] … A 269 (2000) 309 318.

[11] … (… 1995), … 1999, … 168 183.

[12] … 56 (1) (1996) 40 56.

[13] … 64 (3) (1992) 795 848.

[14] … 7 (1997) 139 147.

[15] … 5 (1992) 149 160.

[16] … 2000.

[17] … 134 (1999) 153 184.

[18] … 23 (1989) 74 76.

[19] … 57, … 1979, … 272 301.

[20] … 56 (4) (1984) 737 754.