1. Answer the following for the given graph of a function $f(x)$. Give answers in interval notation where relevant (12 pts):

[redacted]

(a) Identify the domain of f.

Solution:

$\boxed{(\quad 3;4]}$

(b) Identify the the range of f.

Solution:

$\boxed{[\quad 2;0)\ [\ [1;1]}$

(c) Find $(f+f)(\quad 1)$.

Solution:

$(f+f)(\quad 1) = f(\quad 1) + f(\quad 1) = \quad 2 \quad 2 = \boxed{\quad 4}$

(d) Find $f(\quad 3)$ if it exists. If the value does not exist write "DNE."

Solution:

$f(\quad 3):\ \boxed{DNE}$

(e) Solve $f(x) = \quad 1$.

Solution:

$\boxed{x = 1}$

(f) Find $(f \circ f)(1)$.

Solution:

$(f \circ f)(1) = f(f(1)) = f(-1) = \boxed{-2}$.

(g) f is not one-to-one. Briefly explain why this function is not one-to-one.

Solution:

The graph of f does not pass the horizontal line test. For example, $f(-2) = -2$ and $f(-1) = -2$ but $-2 \neq -1$.

(h) Find the x-values where $f(x) \geq 0$. Give your answer in interval notation.

Solution:

$\boxed{[2, 4]}$

(i) Find the net change of $f(x)$ from $x = 0$ to $x = 3$.

Solution:

$f(3) - f(0) = 1 - (-2) = \boxed{3}$

(j) Write down a piecewise-defined function that gives the same graph as $f(x)$.

Solution:

$$f(x) = \begin{cases} -2 & \text{if } -3 < x \leq 0 \\ x - 2 & \text{if } 0 < x < 2 \\ 1 & \text{if } 2 \leq x \leq 4 \end{cases}$$

2. The following are unrelated. (7 pts)

(a) Find the center and radius of the circle that has equation: $x^2 + y^2 \quad 4y = 3$.

Solution:

$$
\begin{aligned}
x^2 + y^2 \quad 4y &= 3 \\
x^2 + y^2 \quad 4y + 4 &= 3 + 4
\end{aligned}
$$

5. For $k(x) = \frac{1}{\sqrt{x}}$ and $j(x) = x^2 + 4$, find the following: (5 pts)

(a) Find $f(x) = (j \circ k)(x)$.

Solution:

$$
\begin{aligned}
(j \circ k)(x) &= j(k(x)) && (12)\\
&= j\left(\frac{1}{\sqrt{x}}\right) && (13)\\
&= \left(\frac{1}{\sqrt{x}}\right)^2 + 4 && (14)\\
&= \boxed{\frac{1}{x} + 4} && (15)
\end{aligned}
$$

(b) Find the domain of $f(x)$.

Solution:

The domain of $f(x)$ must consider both $\frac{1}{\sqrt{x}}$ from line number 13 above and the final expression $\frac{1}{x} + 4$. Thus, the domain of $f(x)$ is $\boxed{(0, \infty)}$.

6. Answer the following for the one-to-one function $h(x)$ whose graph is given below with domain $[0; 2]$. (6 pts)

(a) On the graph to the right, graph the line $y = x$.

Solution:

(b) On the same graph sketch the graph of $h^{-1}(x)$ (label at least two points on the graph of $h^{-1}(x)$).

Solution:

See above graph.

(c) What is the range of $h^{-1}(x)$ in interval notation?

Solution:

The range of $h^{-1}(x)$ is the domain of $h(x)$ which is given in the statement of the problem. So the range of $h^{-1}(x)$ is $\boxed{[0; 2]}$.

7.

(c) $(x - 2)^2 + (y + 1)^2 = 4$

Solution:

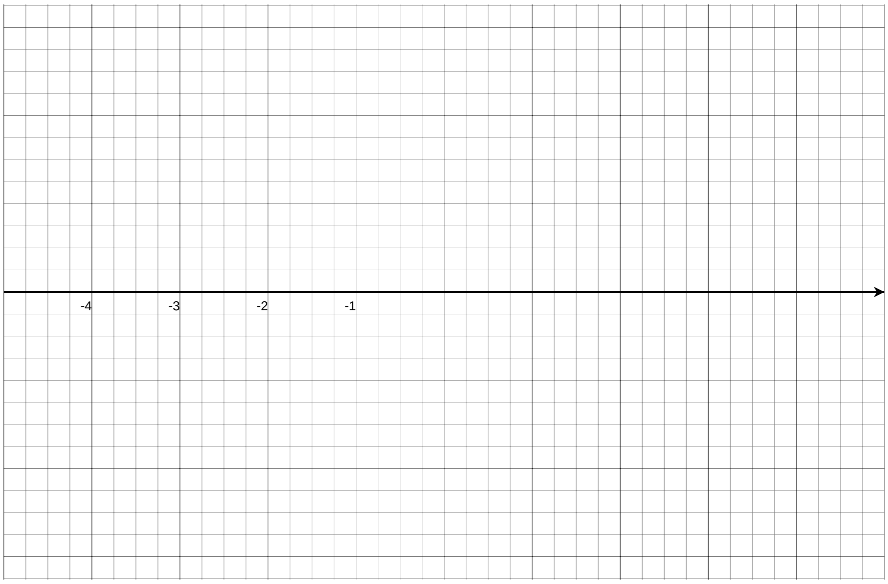
[redacted]

(d) $g(x) = \sqrt[3]{x} + 2$

Solution:

(e) $m(x) = \quad |x|$

Solution:



(f) $q(x) = \begin{cases} 1 & \text{if } x < 1 \\ x^2 \quad 1 & \text{if } x \quad 1 \end{cases}$

Solution:

[redacted]

8. For $P(x) = \quad x^4 \quad 5x^3 \quad 4x^2$ answer the following. (7 pts)

(a) Indicate on a graph or use arrow notation to indicate the end behavior of $P(x)$.

Solution:

For end behavior: $\boxed{P(x) \quad\quad x^4 \;\; ! \quad\; 1 \text{ as } x \;\; ! \;\; 1 \text{ and } P(x) \quad\quad x^4 \;\; ! \quad\; 1 \text{ as } x \;\; ! \quad\; 1}$.

(b) Find the y-intercept of $P(x)$.

Solution:

The y-intercept is found by setting $x = 0$. So $P(0) = \quad 4 \; 0^4 \quad\; 5 \; 0^3 \quad\; 4(0^2) = 0$. So the y-intercept is $\boxed{(0; 0)}$.

(c) Find all zeros and identify the multiplicity of each zero.

Solution:

The zeros of a polynomial are the x-values that result in $P(x) = 0$. So we set $\quad x^4 \quad 5x^3 \quad 4x^2 = 0$. By factoring:

$$x^4 \quad 5x^3 \quad 4x^2 = 0 \tag{16}$$

$$x^2 \;\; x^2 + 5x + 4 \;\; = 0 \tag{17}$$

$$x^2(x+1)(x+4) = 0 \tag{18}$$

So we get $x = 0$ and $x = \quad 1$ and $x = \quad 4$ as the zeros. The multiplicity of $\boxed{x = 0}$ is $\boxed{2}$ and $\boxed{x = \quad 1}$ is $\boxed{1}$ and $x = \quad 4$

10. Use long division to find the quotient and remainder when $2x^3 + 3x^2 - 6x + 2$ is divided by $x^2 - 3$. (5 pts)

Solution:

$$
\begin{array}{r|l}
 & 2x + 3 \\
x^2 - 3 & 2x^3 + 3x^2 - 6x + 2 \\
 & -(2x^3 - 6x) \\
 & 3x^2 + 2 \\
 & -(3x^2 - 9) \\
 & 11
\end{array}
$$

So the quotient is $\boxed{2x + 3}$ and the remainder is $\boxed{11}$.

11. The following are unrelated. (6 pts)

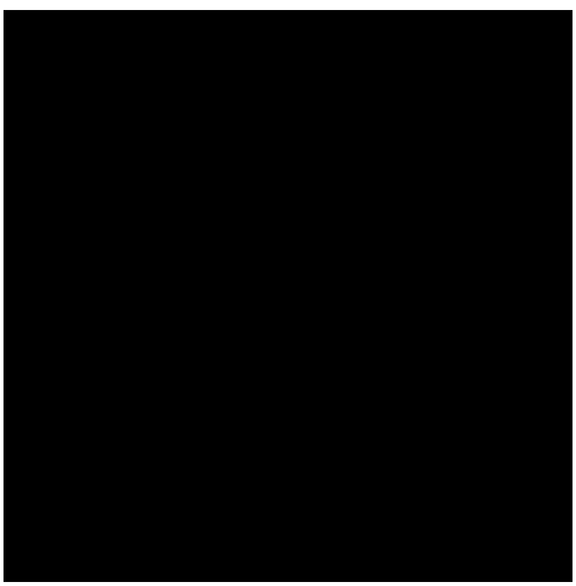
(a) Is $f(x) = x^6 - |x| + 1$ odd, even, or neither? Justify your answer to earn credit.
Solution:

Replacing x by $-x$ and using the fact that $(-x)^6 = x^6$ and $|-x| = |x|$ we get:

$$
\begin{aligned}
f(-x) = &= (-x)^6 - |-x| + 1 &(19)\\
&= x^6 - |x| + 1 &(20)\\
&= f(x) &(21)
\end{aligned}
$$

Since $f(-x) = f(x)$ then $f(x)$ is $\boxed{\text{even}}$.

(b) Is the graph below that of an odd function, even function, or neither?



Solution:

The graph is symmetric about the origin and is thus $\boxed{\text{odd}}$.

12. (a) Plot the points $C(4; 1)$ and $D(4; 3)$ on the graph below. (7 pts)

Solution:

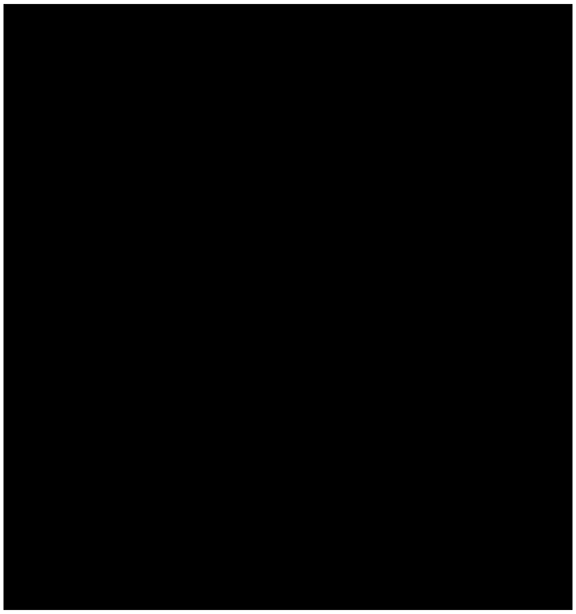
(b) Find the distance between points C and D.

Solution:

The distance between the two points can be found using the distance formula: $d = \sqrt{(4 - 4)^2 + (3 - 1)^2} = \boxed{2}$.

Note that this is the same as simply subtracting the smaller y-coordinate from the larger y-coordinate: $3 - 1 = 2$.

(c) Two flies, Fly A and Fly B, are crawling along a wall in such a way that Fly A is always directly above Fly B (See picture). Fly A is crawling along the path $f(x) = \quad x^2 + \frac{11}{2}x \quad 3$ and fly B is crawling along path $g(x) = x \quad 3$. Find the maximal distance, d, between the two flies on the interval of x-values: $[0; 4:5]$. As always, show all work in justifying your answer.



Solution:

The vertical distance between the two flies can be found by subtracting the smaller y-coordinate from the larger y-coordinate. Letting d represent the distance between the two flies we get:

$$d = \quad x^2 + \frac{11}{2}x \quad 3 \quad (x \quad 3) \tag{22}$$

$$= \quad x^2 + \frac{9}{2}x \tag{23}$$

The x-coordinate, where the maximum distance is located, can be found by using the vertex formula $h = \frac{b}{2a}$ where $a = \quad 1$ and $b = \frac{9}{2}$. Thus $h = \frac{\frac{9}{2}}{2(\quad 1)} = \frac{9}{4}$. The maximum distance is found by plugging in h: $d = \quad \frac{9}{4}^{\;2} + \frac{9}{2} \; \frac{9}{4} = \quad \frac{81}{16} + \frac{81}{8} = \boxed{\frac{81}{16}}$.