Check that the functions are linearly independent

$$W\left(x^{-3}; x\right) = \begin{vmatrix} x^{-3} & x \\ -3x^{-4} & 1 \end{vmatrix} = 4x^{-3} \neq 0$$

so the two functions are linearly independent. $\left\{x^{-3}; x\right\}$ is a basis for the solution space.

(c) We need to use variation of parameters so start by putting the differential equation into standard form

$$y'' + \frac{3}{x}y' - \frac{3}{x^2}y = \frac{1}{x^3}$$

We'll let $y_1 = x^{-3}$ and $y_2 = x$, $f(x) = x^{-3}$ from the differential equation and $W[y_1; y_2] = 4x^{-3}$ from part (b). The particular solution will have the form $y_p = v_1y_1 + v_2y_2$ where

$$v_1' = \frac{-y_2 f}{W[y_1; y_2]} = \frac{-x\,x^{-3}}{4x^{-3}} = -\frac{x}{4} \Rightarrow v_1 = \int -\frac{x}{4}\,dx = -\frac{x^2}{8}$$

$$v_2' = \frac{y_1 f}{W[y_1; y_2]} = \frac{x^{-3}\,x^{-3}}{4x^{-3}} = x$$

(d) (3 pts) If $\quad = 3$ and the mass of the object is 4, what is the value of the spring/restoring constant if the oscillator is critically damped?

SOLUTION:

(a) $x(0) = \quad 1; \dot{x}(0) = 0$

(b) $\quad = 0; \omega_0 > 0; F_0 \neq 0; \quad = \omega_0$

(c) i. yes

ii. infinitely many

iii. yes

(d) Critically damped means $4 \quad ^2 \quad 4\omega_0^2 = 0 \Rightarrow 9 \quad \frac{k}{4} = 0 \Rightarrow k = 36$

5. [2360/041322 (12 pts)] Characteristic equations for certain constant coefficient linear homogeneous differential equations are given along with a forcing function, $f(t)$. Give the form of the particular solution you would use to solve the nonhomogeneous [with the given $f(t)$] differential equation from which the characteristic equation was derived using the Method of Undetermined Coefficients. Do not solve for the coefficients.

(a) (4 pts) $r(3r \quad 1) = 0;\ f(t) = \quad 3 + \sin t$

(b) (4 pts) $r^2 + 2r + 5 = [r \quad (\quad 1 + 2i)][r \quad (\quad 1 \quad 2i)]) = 0;\ f(t) = e^{\quad t} + 5\cos 2t$

(c) (4 pts) $(r \quad 3)^3(r + 2) = 0;\ f(t) = te^{3t} + 2e^{\quad 2t}$

SOLUTION:

(a) Homogeneous solutions are $1; e^{t/3}$; $y_p = At + B\cos t + C\sin t$

(b) Homogeneous solutions are $e^{\quad t}\cos 2t; e^{\quad t}\sin 2t$; $y_p = Ae^{\quad t} + B\cos 2t + C\sin 2t$

(c) Homogeneous solutions are $e^{3t}; te^{3t}; t^2e^{3t}; e^{\quad 2t}$; $y_p = t^3(At + B)e^{3t} + Cte^{\quad 2t}$