

8. [2360/043022 (20 pts)] Consider the linear system of differential equations given by $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$ where $\mathbf{A} = \begin{pmatrix} a & 1 & 1 \\ a & 2 & 1 \end{pmatrix}$ (a is a real number) and with equilibrium solution $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

- Is $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ the only possible equilibrium solution? Justify your answer.
- For what value(s) of a , if any, will the equilibrium solution $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ be a saddle?
- For what value(s) of a , if any, will the equilibrium solution $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ be unstable?
- For what value(s) of a , if any, will the equilibrium solution $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ be asymptotically stable?