

Write your name below. This exam is worth 100 points. On each problem (except for problem 1), you must show all your work to receive credit on that problem. You are NOT allowed to use your notes, book, calculator, or any other electronic devices.

Name: \_\_\_\_\_

1. (21 points: 3 each) If the statement is always true mark `\TRUE`"; if it is possible for the statement to be false then mark `\FALSE`". No justification is necessary.

(a) Let

2. (19 points) Consider the following matrix A

$$A = \begin{pmatrix} 2 & 0 & 3 & 0 & 2 \\ 6 & 1 & 2 & -3 & 0 \\ 6 & 2 & 5 & -4 & 3 \\ -3 & -4 & 7 & 0 & 5 \end{pmatrix}$$

- (a) (7 points) Find the permutation matrix P such that  $B := PA$  is symmetric. Show both P and B.
- (b) (12 points) Can B be factored as  $LDL^T$ ? If yes, find the factorization. If no, justify why it cannot be factored.

3. (20 points: 10 each)

The following two problems are unrelated.

(a) Determine if the following matrices are linearly independent

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}; \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}; \begin{pmatrix} 1 & -5 \\ -4 & 0 \end{pmatrix};$$

(b) Let  $V = \mathbb{R}^4$  and  $W \subseteq V$  be the space spanned by the vectors:

$$\begin{pmatrix} 1 \\ -2 \\ 5 \\ -3 \end{pmatrix}; \begin{pmatrix} 1 \\ 3 \\ 1 \\ -4 \end{pmatrix}; \begin{pmatrix} 1 \\ 8 \\ -3 \\ -5 \end{pmatrix};$$

Find a basis and dimension for  $W$ .

4. (19 points)

The following two questions are unrelated.

(a) (9 points) Let  $V = \mathbb{R}^3$  and  $W = ($

5. (21 points)

$$\text{Let } A = \begin{pmatrix} 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 4 & 3 & 4 & 1 & 1 \\ 1 & 2 & 2 & 2 & 1 & 1 \end{pmatrix} A.$$

- (3 points) What is the rank of  $A$ ?
- (3 points) What is  $\dim \text{coker } A$ ?
- (5 points) Find a basis for the image of  $A$ .
- (5 points) Find a basis for the coimage of  $A$ .
- (5 points) Find a basis for the kernel of  $A$ .