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## abstract

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$> 0$, $\delta > 0$, $r^{-\beta}$, $\beta > 0$,

$$\left| r^{-\beta} - \sum_{m=1}^{M} w \right.$$

**1.** *Let us assume that* (4) *holds. For any* $> 0$ *and* $t_0 \in \mathbb{R}$, *we have*

$$\left| \int_{\mathbb{R}} f(t)\,dt - h \sum_{n \in \mathbb{Z}} f(t_0 + nh) \right| \leqslant \tag{6}$$

*provided that the Fourier transform of* $f$ *satisfies*

$$\left| \hat{f}(\xi) \right| \leqslant c_1 e^{-q|\xi|}, \tag{7}$$

*for some positive constants* $c_1$, $q$ *and step size* $h \leqslant q/$ $(2c_1 \;\; ^{-1} + 1)$ *or, alternatively,*

$$\left| \hat{f}(\xi) \right| \leqslant \frac{c_2}{|\xi|^q}, \quad \text{for } |\xi| \geqslant R, \tag{8}$$

*for some positive constants* $c_2$, $R$, $q$ *and step size* $h \leqslant$ $\{1/R, \;\; ^{1/q}(2c_2\zeta(q))^{-1/q}\}$, *where* $\zeta(q)$ *is the Riemann Zeta function.*

(5), $h$ $\sum_{n\neq 0} |\hat{f}(\frac{n}{h})| \leqslant$ (7),

$$\sum_{n \neq 0} \left| \hat{f} \; \{ \right.$$

$$S_\infty(r) = \frac{h}{\ (\beta)} \sum_{n\in\mathbb{Z}} e^{\beta(t_0+nh)} e^{-e^{t_0+nh} r}. \tag{13}$$

$h = h(\ ,\beta)$

$$\sum_{n\neq 0} \frac{|\ (\beta + 2\pi i \frac{n}{h})|}{\ (\beta)} < \ . \tag{14}$$

**3.** *Given* $\beta > 0$ *and* $0 < \ \leqslant 1$, *for any step size* $h$ *such that*

$$h \leqslant \frac{2\pi}{\ 3+\beta\ (\ 1)^{-1} + \ ^{-1}}, \tag{15}$$

*and any* $t_0 \in \mathbb{R}$ *we have*

$$\frac{|r^{-\beta} - S_\infty(r)|}{r^{-\beta}} \leqslant \ , \quad \text{for all } r > 0, \tag{16}$$

*where* $S_\infty$ *is given in* (13).

$S_\infty(r)$ (16) $r$. ,

$\beta$,3 9 8 ( ) 4 0 .

**4.** *For all* $r > 0$,

$$S_F(r) < S_\infty(r) < ( \;+1)r^{-\beta}.$$

[illegible], $S_F(r)$, [illegible] *on the whole positive axis*, [illegible] 12 [illegible] 4 [illegible] 9 .

[illegible] fi [illegible] $f$ [illegible] (10) [illegible] $r$ [illegible] $\beta$. [illegible] (16). □

[illegible] $S_F$ [illegible] $\beta$, $\delta$ [illegible]

**5.** *For any* $\beta > 0$, $\delta > 0$, *and* 1 [illegible] /_7 1 [illegible] 9.7304 0 0 9.7304 303 *ou8309o94 0 TD 0.2518 f4p -33.1058 -2.866 TD 0.0004 0 9.*

B $M$ $N$ $r \in [\delta, 1]$. $f_{\beta,r}(y)$ (9) (10) $y_0 =$ $(r^{-1}\beta)$. $M$ $t_M \leqslant$ $\beta$ $f_{\beta,r}(y)$ $(-\infty, t_M)$ $r \in (0, 1]$ $\mathcal{T}_M(r)$

$$\mathcal{T}_M(r) \leqslant \frac{r^\beta}{(\beta)} \int_{-\infty}^{t_M} e^{-re^y+\beta y}\, dy \leqslant \frac{1}{(\beta)} \int_{-\infty}^{t_M} e^{-e^y+\beta y}\, dy \tag{27}$$

$$= \frac{1}{(\beta)} \int_0^{e^{t_M}} e^{-s} s^{\beta-1}\, ds = 1 - \frac{(\beta, e^{t_M})}{(\beta)}, \tag{28}$$

$$(\beta, x) = \int_x^\infty e^{-s} s^{\beta-1}\, ds$$

$t_M$ $\delta$.

$N$

$$t_N \geqslant \left(\delta^{-1}\beta\right), \tag{29}$$

fi

$$\mathcal{T}^N(r) \leqslant \frac{r^\beta}{(\beta)} \int_{t_N}^{\infty} e^{-re^y+\beta y}\, dy = \frac{1}{(\beta)} \int_{re^{t_N}}^{\infty} e^{-s} s^{\beta-1}\, ds,$$

$r \in [\delta, 1]$

$$\mathcal{T}^N(r) \leqslant \frac{(\beta, \delta e^{t_N})}{(\beta)}. \tag{30}$$

$$\lim_{x \to 0} \frac{(\beta, x)}{(\beta)} = 1 \qquad \lim_{x \to \infty} \frac{(\beta, x)}{(\beta)} = 0,$$

fi $t_*$ $t^*$

$$1 - \frac{(\beta, e^{t_*})}{(\beta)} = \ , \tag{31}$$

$$\frac{(\beta, \delta e^{t}\ )}{(\beta)} = \ . \tag{32}$$

$M_*$ $N^*$ fi $S_F$, $t_*$, $t^*$, $h$ $t_0$, $(t_* - t_0)/h$ $(t^* - t_0)/h$

**7.** *For all* $\beta > 0$, $\delta > 0$ *and* $1/e \geqslant \ > 0$, *the solution* $t_*$ *of* (31) *does not depend on* $\delta$ *and satisfies*

$$t_* \geqslant \frac{(1+\beta)}{\beta} = \frac{1}{\beta} \quad + \quad (1+\beta)^{\frac{1}{\beta}}. \tag{33}$$

*The solution* $t^*$ *of* (32) *has a weak dependence on* *and satisfies*

$$t^* \leqslant \ \delta^{-1} + \quad {}^{-1} + \ \beta + \frac{1}{2}. \tag{34}$$

$t_*$ $t^*$, (33) (34). $S_F$ 5. 7 A A.2. (9) (10) $-\infty$ $+\infty$ $S_F$ $e^{hn}$ (21) ( $n$). A 16 (21) 3 .

**8.** *For any $\delta > 0$, and $> 0$, there exist a step size $h$ and a positive integer $M$ such that*

$$\left|e^{-xy} - G_e(x, y)\right| \leqslant \quad , \quad \text{for } xy \geqslant \delta, \tag{41}$$

*where*

$$G_e(x, y) = \frac{hx}{2\sqrt{\pi}} \sum_{j=0}^{M} e^{-\ x^2}$$

$\leqslant$

*be an approximation of the kernel by Gaussians valid for $\delta \leqslant r \leqslant 1$. Then, for any bounded, compactly supported function $f$ in $D$ and $x \in D$, we have*

$$\left| \int_{B_1} \|\ \|^{-\alpha} f(\ \ +\ )\, d\ - \int_{B_1} G_F\big(\|y\|\big) f(\ \ +\ )\, d\ \right| \leqslant \big(\ \ + (2+\ \ )\delta^{d-\alpha}\big) \frac{\omega_{d-1}}{d-\alpha} \|f\|_\infty.$$

. C $B_\delta$

A $u(z,\;)$ $z>0$, $u_{zz}+\;u=0$ $u(0,\;)=u_0(\;)$,

$$u(z,\;)=\int_{\mathbb{R}^d}\mathcal{P}(z,\;-\;)u_0(\;)\,d\;,\quad z\geqslant 0, \tag{56}$$

$$\mathcal{P}(z,\;)=\frac{2}{\omega_d}\frac{z}{(z^2+\|\;\|^2)^{(d+1)/2}}$$

0 3, $>0$ $z>0$, 0

$$S_\infty\big(z^2+\|\;\|^2\big)=\frac{zh}{\pi^{(d+1)/2}}$$

$0 \leqslant n \leqslant 2N$. , $a > 0$ [0, 1] ffi $w_m$ (66) $\eta_m = \frac{2N}{a} t_m$, $h(x)$ $x \in [0, a]$,

$$\left| h(x) - \sum_{m=1}^{M} w_m e^{-\eta_m x} \right| < \ ', \tag{67}$$

$h$ (66) . (66) :

- B $(N+1) \times (N+1)$ 0 $_{kl} = h_{k+l}$ $h_n = h(a\frac{n}{2N})$, $0 \leqslant n \leqslant 2N$.
- $= (u_0, \dots, u_N)$, $= \sigma$ , $\sigma$ 0 ( 16, . 22 ); $\sigma$ fi $M+1$ $\sigma_0 \geqslant \sigma_1 \geqslant \dots \geqslant \sigma_M$, $\sigma_M$ $\sigma_M/\sigma_0 \approx$ , , $M = \mathcal{O}( \ ^{-1})$ $M \ll N$.
- C $\gamma_m$ $u($

$x = {}^{-1}$. , $\beta$

$$\tilde{t}^* = \delta^{-1} + {}^{-1}. \tag{70}$$

$\beta > 1$,

$$\frac{(\beta, x)}{(\beta)} \leqslant \frac{e}{(\beta)} x^{\beta-1} e^{-x} \leqslant ,$$

fi (69)

$$x > d(\beta - 1) \tag{71}$$

$d = e/(e-1) \approx 1.582$, , 14, $x$

$$x \geqslant d$$

*A.3. Proof of Theorem 5*

2.1.1 (20) $h$ (15) $t_*$ $t^*$ 7. , $t_0 = 0$

$$\tilde{h} = \frac{10}{2 \quad ^{-1} + \quad \beta + 2}.$$

,

$$\frac{\tilde{t}^* - \tilde{t}_*}{\tilde{h}},$$

$\tilde{t}^*$, $\tilde{t}_*$ (33) (34).

$$\tilde{t}^* - \tilde{t}_* = \quad \delta^{-1} + \frac{1}{\beta} \quad ^{-1} + \quad ^{-1} + \left( \frac{\beta+1}{(1+\beta)^{\frac{1}{\beta}}} \right) + \frac{1}{2}$$

$$\leqslant \quad \delta^{-1} + \frac{1}{\beta} \quad ^{-1} + \quad ^{-1} + \frac{3}{2}$$

15.

(23) (27) (28).

**15.** *Let* $g(x) = \frac{(x+1)^{\frac{1}{x}}}{x+1}$ *for* $x > 0$

33 B 0 : A 0 C 121 (7) (2004) 2866 2876.

34 B 0 : 0 C 121 (14) (2004) 6680 6688.

35 0 A C 20 (2) (1999) 699 718 ( ).