Mathematical Geophysics Summer School, Stanford, August 1998

Wavelet transforms and compression of seismic data

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I Introduction

Seismic data compression (as it exists today) is a version of transform coding which involves three main steps:

1. The transform step, which is accomplished by a fast wavelet, wavelet-packet or local cosine transform;
2. The quantization step, which is typically accomplished by a scalar uniform or non-uniform quantization scheme, and
3. The encoding step, which is accomplished by entropy coding such as Huffman coding or adaptive arithmetic coding.

Let us briefly outline the role of each step. The role of the transform step is to decorrelate the data. Namely, the transform will take a data set with a more or less flat histogram and produce a data set which has just a few large values and a very high number of near zero or zero values. In short, this step prepares the data for quantization. It has been observed that a much better compression is achieved by quantizing the decorrelated data than the original data.

There are several transforms that can be used for decorrelation. For example, the Karhunen-Loeve transform achieves decorrelation but at a very high computational cost. It turns out that the wavelet, wavelet-packet or local cosine transforms can be used instead. These transforms are fast and provide a local time-scale (or time-frequency) data representation, resulting in a relatively few large coefficients and a large number of small coefficients.

At the second step the coefficients of transformed data are quantized, i.e., mapped to a discrete data set. The quantization can take two forms, either scalar quantization, or vector quantization. In the case of scalar quantization every transform coefficent is quantized separately whereas in the case of vector quantization a block of coefficients is quantized simultaneously. Based on practical experience with seismic data it appears that the transformed coefficents tend to be reasonably decorrelated, thus pointing to scalar

A distortion criterion (implied by the size of the seismic gather or ensemble of gathers and the target compression ratio) is minimized subject to the bit budget. In some cases a non-uniform quantization can yield lower distortion level than the uniform quantization.

We note that some new quantization/coding schemes which have been used for image compression may not be directly applicable to seismic data. For example, embedded zero-wavelet tree compression (EZW) scheme [13] does not appear e cient since seismic data violate the basic assumptions of EZW algorithm.

After quantization we are likely to have a number of repeated quantized coe cients and, thus, a signi cant redundancy. The third step, entropy coding, addresses this issue. Perhaps the easiest analogy to entropy coding comes from the Morse code communication, in which frequently encountered symbols are transmitted with shorter codes, while rarely encountered symbols are transmitted with longer codes. The entropy coding creates a new data set which has the average number of bits/sample minimized. There are two distinct cases of entropy coding. In the case of stationary data one can use Hu man coding. In the case of non-stationary data adaptive arithmetic coding is usually applied.

Now that we have an overall picture, we will desribe individual steps in greater detail. The notions that are considered below are not yet a familiar territory for a geophysicist and, for that reason, our goal will be limited to providing a basic trail map. We will consider the basic steps in the reverse order so that it is clear (at least intuitively) what is desirable to have as an output of the preceding step.

II Entropy coding

Let us consider a nite set of symbols $S = (x_1; x_2; :::; x_N)$. Let p_n denote the propability of occurence of the symbol x_n in some set X (elements of which are from S), $\sum_{n=1}^{N} p_n$.Theva Tj 10.ote

The Hu man algorithm (Hu man coding) [7] constructs an optimal pre x tree and the corresponding code so that

$$H(X) \quad R_X \quad H(X) + 1:$$

The di culty in obtaining the lower bound via Hu man coding is that it requires

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for $j = 0; \ldots; n - 1$ and $k = 0; \ldots; 2^{n-j-1} - 1$. It is easy to see that evaluating the whole set of coefficients d_k^j, s_k^j in (5.5), (5.6) requires $2(N - 1)$ additions and $2N$ multiplications.

In two dimensions, there are two natural ways to construct the Haar basis. The first is simply the tensor product

Let us consider a multiresolution analysis for $L^2(R)$ and let f (x k

Computing via (5.28) and (5.29) is illustrated by the pyramid scheme

$$\begin{array}{ccccccccccc} \{s_k^0\} & \rightarrow & \{s_k^1\} & \rightarrow & \{s_k^2\} & \rightarrow & \{s_k^3\} & & \\ & \searrow & & \searrow & & \searrow & & & \\ & & \{d_k^1\} & & \{d_k^2\} & & \{d_k^3\} & & . \end{array} \tag{5.30}$$

The reconstruction of a function from its wavelet representation is also an order N procedure and is described by

$$\begin{aligned} s_{2n}^{j-1} &= \sum_{k=1}^{k=M} h_{2k} s_{n-k}^{j} + \sum_{k=1}^{k=M} g_{2k} d_{n-k}^{j}, \\ s_{2n-1}^{j-1} &= \sum_{k=1}^{k=M} h_{2k-1} s_{n-k}^{j} + \sum_{k=1}^{k=M} g_{2k-1} d_{n-k}^{j}. \end{aligned} \tag{5.31}$$

Computing via (5.31) is illustrated by the pyramid scheme

$$\begin{array}{ccccccc} \{s_k^n\} & \rightarrow & \{s_k^{n-1}\} & \rightarrow & \{s_k^{n-2}\} & \rightarrow & \{s_k^{n-3}\} \\ & \nearrow & & \nearrow & & \nearrow & \\ \{d_k^n\} & & \{d_k^{n-1}\} & & \{d^{n-2} \;\; 2 \;\; 2\, 2 & & \end{array}$$

VI What does wavelet transform accomplish?

Although the following consideration deals with wavelet transform, similar points can be made for wavelet packets and local trigonometric bases.

The coherent portion of the signal in a seismogram appears as a local correlation that our eye easily identi es. Wavelet transform reduces the number of signi cant coe cients necessary to represent the seismogram locally and, thus, decorrelates the coherent portion of the signal. Vanishing moment are the key to such performance. On the other hand the wavelet transform does not decorelate (or compress) the random Gaussian noise. Thus, the result of application of the wavelet transform is that the coherent portion of the signal will now reside in a relatively few large coe cients whereas the rest of the coe cients will describe a portion of the signal that is like the \random Gaussian noise". Unfortunately, this heuristics is not precise and there is no theorem asserting the result. Yet, a number of mathematically justi ed statements arestatemen

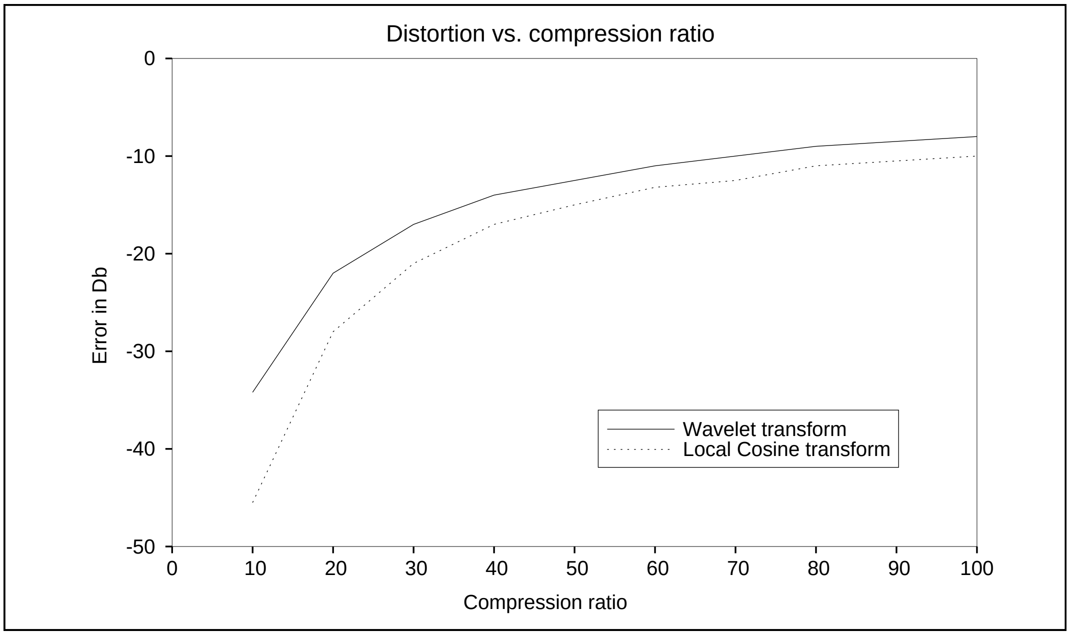

Figure 1: Distortion Curve: wavelet vs. local cosine transform for land data

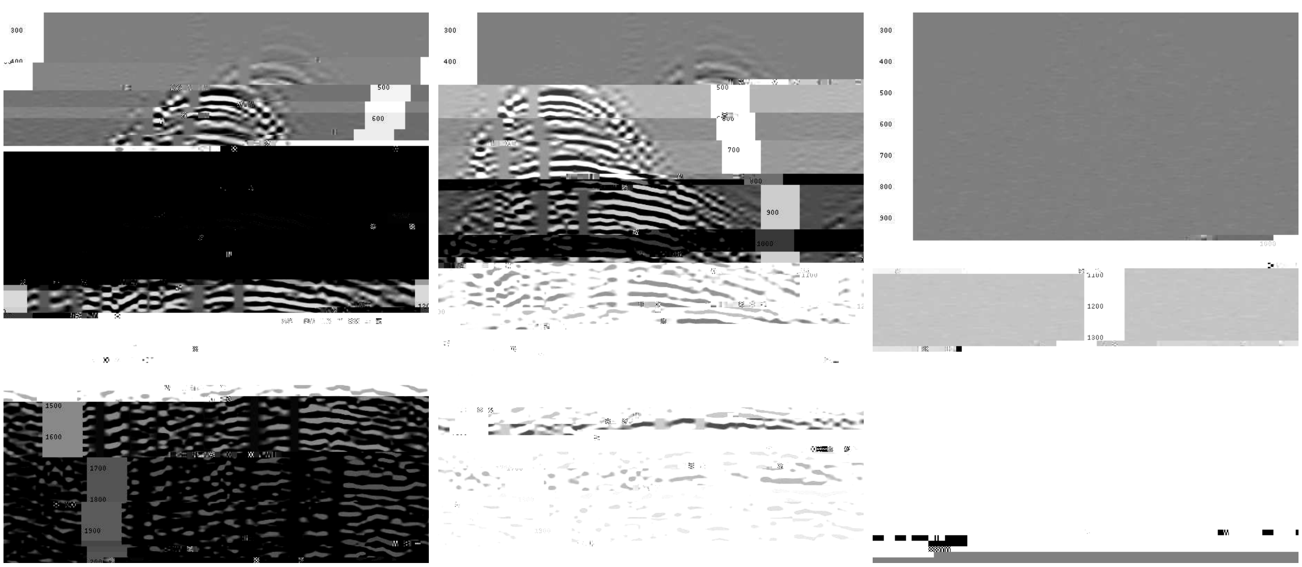

Figure 2: Land data: original, compressed by the factor of

Compression ratio

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