

Factorial growth of the number of states in a quantum system

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FIG. 1. () N
 $-1/2$ R
 $= N / 2$

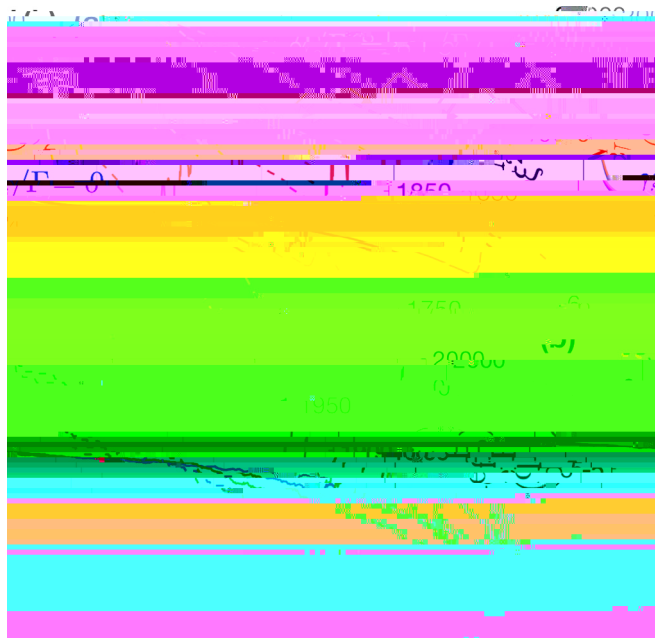


FIG. 2. (a) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 2$. (b) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 4$. (c) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 8$. (d) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 16$. (e) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 32$. (f) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 64$. (g) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 128$. (h) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 256$. (i) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 512$. (j) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 1024$. (k) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 2048$. (l) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 4096$. (m) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 8192$. (n) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 16384$. (o) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 32768$. (p) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 65536$. (q) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 131072$. (r) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 262144$. (s) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 524288$. (t) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 1048576$. (u) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 2097152$. (v) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 4194304$. (w) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 8388608$. (x) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 16777216$. (y) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 33554432$. (z) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 67108864$. (aa) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 134217728$. (ab) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 268435456$. (ac) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 536870912$. (ad) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 1073741824$. (ae) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 2147483648$. (af) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 4294967296$. (ag) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 8589934592$. (ah) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 17179869184$. (ai) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 34359738368$. (aj) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 68719476736$. (ak) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 137438953472$. (al) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 274877906944$. (am) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 549755813888$. (an) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 1099511627776$. (ao) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 2199023255552$. (ap) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 4398046511104$. (aq) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 8796093022208$. (ar) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 17592186044416$. (as) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 35184372088832$. (at) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 70368744177664$. (au) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 140737488355328$. (av) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 281474976710656$. (aw) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 562949953421312$. (ax) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 1125899906842624$. (ay) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 2251799813685248$. (az) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 4503599627370496$. (ba) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 9007199254740992$. (bb) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 18014398509481984$. (bc) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 36028797018963968$. (bd) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 72057594037927936$. (be) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 144115188075855872$. (bf) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 288230376151711744$. (bg) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 576460752303423488$. (bh) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 1152921504606846976$. (bi) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 2305843009213693952$. (bj) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 4611686018427387904$. (bk) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 9223372036854775808$. (bl) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 18446744073709551616$. (bm) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 36893488147419103232$. (bn) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 73786976294838206464$. (bo) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 147573952589676412928$. (bp) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 295147905179352825856$. (bq) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 590295810358705651712$. (br) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 1180591620717411303424$. (bs) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 2361183241434822606848$. (bt) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 4722366482869645213696$. (bu) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 9444732965739290427392$. (bv) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 18889465931478580854784$. (bv) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 37778931862957161709568$. (bv) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 75557863725914323419136$. (bv) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 151115727451828646838272$. (bv) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 302231454903657293676544$. (bv) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 604462909807314587353088$. (bv) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 1208925819614629174706176$. (bv) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 2417851639229258349412352$. (bv) $N = 2000$, $\beta = 0$, $\beta_c = 0.9$, $N_c = 4835703278458516698824704$. 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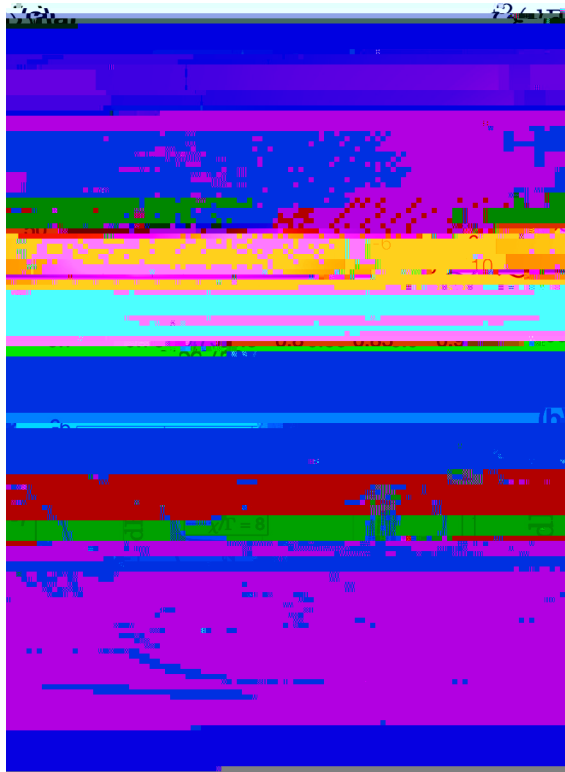


FIG. 3. () M_{eff} vs t/c for $\beta = 0.9$ and $\beta = 0.95$. The plot shows the effective mass M_{eff} as a function of t/c for two different values of β . The data points are shown as black dots with error bars, and the solid lines represent fits to the data. The $\beta = 0.9$ data (top) shows a peak around $t/c \approx 10$, while the $\beta = 0.95$ data (bottom) shows a peak around $t/c \approx 15$. The $N = 10^4$ is indicated in the caption.

R. $\beta = 0.9$ and $\beta = 0.95$. The plot shows the effective mass M_{eff} as a function of t/c for two different values of β . The data points are shown as black dots with error bars, and the solid lines represent fits to the data. The $\beta = 0.9$ data (top) shows a peak around $t/c \approx 10$, while the $\beta = 0.95$ data (bottom) shows a peak around $t/c \approx 15$. The $N = 10^4$ is indicated in the caption.

