

with $x_j(t) = x_j$ and will show $\lim_{t \rightarrow \infty} |x_1(t) - x_2(t)| = 0$. At $O(\epsilon)$, we find Eq. (2) with a corresponding linear operator $Lu = \epsilon u + w[f(U) \cdot u]$. Solvability is enforced by ensuring the right-hand side of Eq. (2) is orthogonal to the null space V of the adjoint $L^*p = \epsilon p + f(U) \cdot w(\epsilon p)$, yielding the Langevin equation, Eq. (4). The Lyapunov exponent associated with the stability of the absorbing state $x_1(t) = x_2(t)$ is then approximated by Eq. (14).

To compare our analytical results for traveling waves with numerical simulations, we compute from Eq. (14) when $f(u) = H(u - x)$, $w(x) = \cos(x - x)$, and $C(x) = \cos(x)$. Stable traveling waves have a profile $\phi(x) = \cos[\sin(x - x) + a]$, width $a = \sin^{-1}[\text{sec}]$ defined by thresholds $\phi(x_1) = U(x_2) = \text{where } x_1 = x - a$ and $x_2 = x$, and speed $v = \tan[\text{sec}]$ [28]. The null vector can also be computed explicitly:

$$V(x) = \sum_{k=1}^2 (\sin)^k H(x - x_k) + \frac{\coth(x/c) - 1}{2} e^{(kx)/c}.$$

Fourier coefficients of $W(x, t)$ in Eq. (6) are thus given $b_{\pm 1} = (1 - c)/$

so $\langle W_\ell(\mathbf{x}) \rangle = 0$; $\langle W_\ell(\mathbf{x}) W_\ell(\mathbf{x}') \rangle = 2C_\ell(\mathbf{x} - \mathbf{x}') \delta_{\ell c}$ ($\ell = 1, 2, c$) with $C_\ell(\mathbf{x}) = \int_{-\infty}^{\infty} a \cos(\mathbf{x} \cdot \mathbf{q})$. The degree of correlation between layers is controlled by the parameter μ .

Our analysis proceeds by considering stationary bumps in a network with even symmetric connectivity [$\mathcal{J}(\mathbf{x}) = \mathcal{J}(-\mathbf{x})$]. As in the main text, we characterize stochastic bump motion by applying the ansatz $\varphi_\ell(\mathbf{x}) = U(\mathbf{x} - \Delta_\ell(\mathbf{x})) + \varepsilon \Phi_\ell(\mathbf{x} - \Delta_\ell(\mathbf{x})) + O(\varepsilon^2)$, and $\Delta_\ell(0) = \mathbf{0}$. Plugging this ansatz into Eq. (A1), expanding to $O(\varepsilon)$, and applying a solvability condition, we find that each Δ_ℓ ($\ell = 1, 2$) obeys the Langevin equation

$$\Delta_\ell = \varepsilon, \frac{-\nabla V(\mathbf{x}) \cdot \nabla \mathcal{J}(\mathbf{x})}{-\nabla V(\mathbf{x})}$$

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