

Cluster synchrony in systems of coupled phase oscillators with higher-order couplingPe Seba¹, Ian Ska^{1,*}, Ed a d O², and J an G. Re e o¹¹*Department of Applied Mathematics, University of Colorado at Boulder, Colorado 80309, USA*²*Institute for Research in Electronics and Applied Physics, University of Maryland, College Park, Maryland 20742, USA*

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We study the phenomenon of cluster synchrony that occurs in ensembles of coupled phase oscillators when higher-order mode dominates the coupling between oscillators. For the first time, we develop a complete analytic description of the dynamics in the limit of a large number of oscillators and derive an analytical expression for the degree of cluster synchrony, cluster asymmetry, and clustering. We establish a relation of the even-dimensional reduction technique of Ott and Anagnostou [Chaos **18**, 037113 (2008)] and find an analytic description of the degree of cluster synchrony valid on a globally attracting manifold. Shaded by this manifold, the eigenvalues in the family of eigenvalues describing the stability of oscillators, leading to a high degree of multistability in the cluster asymmetry. We also show how the external forcing the degree of asymmetry can be controlled, and suggest that the emerging clustering can be used to encode and decode data.

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I. INTRODUCTION

Large ensembles of coupled oscillators occur in many e-

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in the and natural frequency ω at time t . Since the oscillation is a real function of time, the condition for a stationary solution $\partial_t f + (f \cdot) = 0$, giving

$$\partial_t f + \left\{ f \left[\omega + \frac{K}{2i}(r_2 e^{-2i} - r_2^* e^{2i}) \right] \right\} = 0. \quad (6)$$

To analyze Eq. (6), we find it convenient to define the symmetric and antisymmetric parts of f , f_s , and f_a , as

$$f_{s/a}(\mathbf{r}, t) = [f(\mathbf{r}, t) \pm f(\mathbf{r} + \mathbf{a}, t)]/2, \quad (7)$$

where f_s and f_a are symmetric and antisymmetric in the exchange of sites \mathbf{r} and $\mathbf{r} + \mathbf{a}$, respectively, in the sense that $f_s(\mathbf{r} + \mathbf{a}, t) = f_s(\mathbf{r}, t)$ and $f_a(\mathbf{r} + \mathbf{a}, t) = -f_a(\mathbf{r}, t)$. We note that a solution of Eq. (6) if $f = f_s + f_a$ and f_s and f_a are both solutions of Eq. (6). Thus, we can study separately the symmetric and antisymmetric dynamics of solution f .

A. Symmetric dynamics

While the amplitudes r_i and frequencies f_a remain the same. Thus, the only change is in $|r_1|$

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