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Ensemble-based estimates of eigenvector error for empirical covariance matrices

Dane Taylor
14260,
2770 ,
275 ,
orr o g or. m : ff o.

Juan G. Restrepo
8030 ,
g 4o or o.

and

François G. Meyer
8030 ,
fm r 4o or o.

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o r 4 m r 4 r fi m o for 4 of 4o om 4, 4 o og 4

1. Introduction

4 r ro r of 4o r 4 m r 4 r 4 r o 4 m m 4 , ro 4 (, 1991 r o , 2003 , 2009 o & o , 2012) ro 4or r-

o . or m, o o from (1.5) 4 g g o

$$E[\ ]+$$

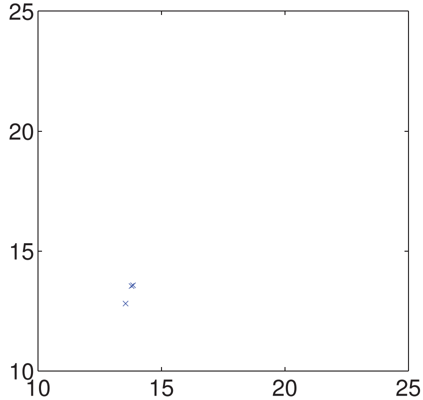
Assumption 2.2 [illegible] m [illegible] o ro [illegible] r o (,) of [illegible] f r g g , + λ λ 1 , ro 4 g λ g fo o g g r g r m for [illegible]

$$(\; , \;) \quad \frac{3^7 [\; \rho(\lambda\;)]^5}{32\pi^3} [\; , \; \langle$$

2.3 2

o (2.2) g m o 4 m for

o o 4 (r ., 2017 T or ., 2017). fi r ork, o r g o o for gr mor 4om 4 n 4 r of 4 4om ork r rg o of ork or g 4 r for gr (rk ., 2001 o ., 2001 g ., 2003 orogo ., 2003 B 4- org & k , 2011 o o, 2013 Z g ., 2014 T or ., 2016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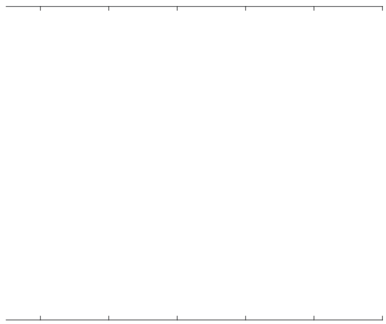

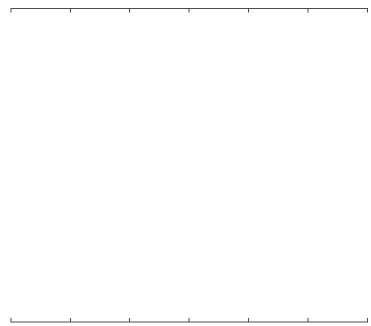

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A. Derivation of main result 1

A. , ro m (1.1) m of r - g o r g g . B o g o, o o k o g of , m g r o of g . g g mm o o o r o

$$+ \,, \tag{A.1}$$

$$+ \sum_{+1}^{1} \frac{\lambda\,\lambda}{(\lambda \quad \lambda\,)^2}, \tag{A.2}$$

$$+ \sum_{+,1} \frac{\lambda\,\lambda}{(\lambda \quad \lambda\,)^2}. \tag{A.3}$$

r m r 4 r m o 4 r - g o r m om o r . T k g o 44o , o r 4 g r r

$$+ \frac{\lambda\,\lambda_{\;1}}{(\lambda \quad \lambda_{\;1})^2} \sum_{+1}^{2} \frac{\lambda\,\lambda}{(\lambda \quad \lambda\,)^2}, \tag{A.4}$$

$$+ \frac{\lambda\,\lambda_{\;1}}{(\lambda \quad \lambda_{\;1})^2} \sum_{+,2} \frac{\lambda\,\lambda}{(\lambda \quad \lambda\,)^2}. \tag{A.5}$$

rg o r of (A.4) (A.5) r 4o r g r - g o r m mm o . r 4 r, o ro m o r o o r g

g 0 , (A. m o 2.2 4o rg 4 ,
4 o , r + (1/)), m o 4 m

$$\frac{\lambda \lambda_{1}}{(\lambda \quad \lambda_{1})^2} \qquad \frac{\lambda^2}{(\quad)^2}. \tag{A.9}$$

o r or o o mm o , ro m g m g
4 r $\rho(\lambda)$ of orm m r 4 4o g m r of g . or
r 4 , 4o r 4 of - mm r 4 4o r 4 m r 4 , g g λ, for
1, . . . , . for 4 m r m r 4 4 r

$$\rho(\lambda) + \quad {}^{1} \sum \delta(\lambda), \tag{A.10}$$

r $\delta(\lambda)$ 4 fi 4 o λ R. m 4o r 4 m r 4 r r from
m 4 k 4o rg , m g

$$\int \rho(\lambda)(\lambda)\ \lambda \qquad \int \rho(\lambda)(\lambda)\ \lambda \tag{A.11}$$

for 4o fi 4 o (λ). m $\rho(\lambda)$ 4o ,
, 4om 4 or (o (ρ)) ff r o (ρ). or o o
4o 4 , m $(\rho) + (\alpha, \beta)$ for om α, β R, o g o r 4 m of
gr o (A.11) (α, β). o r, r 4 o o of r .
g r r g mm o (A.4) (A.5) gr o of fi 4 o

$$(\lambda) + \frac{\lambda \lambda}{(\lambda \quad \lambda)^2} \tag{A.12}$$

ro m r $\rho(\lambda)$ g (A.10),

$$\frac{1}{\ } \sum_{+1}^{2} \frac{\lambda \lambda}{(\lambda \quad \lambda)^2} + \int_{\alpha}^{\lambda_{1}} \rho(\lambda)(\lambda)\ \lambda, \tag{A.13}$$

$$\frac{1}{\ } \sum_{+\ 2\lambda}$$

o r o r g of (A.15) 4 ε. g o $\rho(\lambda)$ (λ) r ff r for λ (ρ) $\|$ λ, m m gr o r , r g m r or om or r , o o

$$\int_{\alpha}^{\lambda \quad \varepsilon} (\lambda)\rho(\lambda) \quad \lambda \; + \; \lambda \frac{(\lambda \quad \varepsilon)\rho(\lambda \quad \varepsilon)}{\varepsilon} \quad \lambda \int_{\alpha}^{\lambda \quad \varepsilon} \frac{\rho(\lambda) \quad \lambda\rho \; (\lambda)}{\lambda \quad \lambda} \quad \lambda. \tag{A.16}$$

r m r g of (A.16) ε 0 m o 4 m

$$\lambda \frac{(\lambda \quad \varepsilon)\rho(\lambda \quad \varepsilon)}{\varepsilon} \quad \frac{\lambda^2 \rho(\lambda)}{\varepsilon}. \tag{A.17}$$

4o m o r g of (A.16) o

$$\left| \lambda \int_{\alpha}^{\lambda \quad \varepsilon} \frac{[\rho(\lambda) \quad \lambda\rho \; (\lambda)]}{\lambda \quad \lambda} \quad \lambda \right| \leqslant \lambda \left[\quad \rho(\lambda) \quad \lambda\rho \; (\lambda) \right] \int_{\alpha}^{\lambda \quad \varepsilon} \frac{1}{\lambda \quad \lambda} \quad \lambda$$

$$+ \lambda \left[\quad \rho(\lambda) \quad \lambda\rho \; (\lambda) \right] \left(\frac{\lambda \quad \alpha}{\varepsilon} \right). \tag{A.18}$$

fo o 4o m r g of (A.16) 4 g ($(1/\varepsilon)$) om m ε 0 r m, 4 $(1/\varepsilon)$. 4om (A.17) (A.18) o o ε 0 m o 4 m

$$\int_{\alpha}^{\lambda \quad \varepsilon} (\lambda)\rho(\lambda) \quad \lambda \quad \frac{\lambda^2 \rho(\lambda)}{\varepsilon}. \tag{A.19}$$

o 4 r $\rho\,(\lambda)$ o , r g for r o r gr o f of (A.18) o o om : o 4o g λ, r $\rho\,(\lambda)$ o 4o om g r m λ ε. r gr 4o rg o ro o (A.11) r 4o o g (A.18), m g gr m (A.16) ($(1/\varepsilon)$), ro $\rho(\lambda)$ ff r m g o r oo 4o g λ. m g o r for r g of (A.13) 4o r g fo o g :

$$\int_{\alpha}^{\lambda \quad 1} (\lambda)\rho(\lambda) \quad \lambda + \int_{\alpha}^{\lambda} (\lambda)\rho(\lambda) \quad \lambda \quad \int_{\alpha}^{\lambda} (\lambda) \left[\rho(\lambda) \quad \rho(\lambda) \right] \quad \lambda$$

mm r , 4om (A.21), (A.22) (A.17

To o m for r o of ( ), ff r (B.6) r 4 o o o

$$(\cdot) + \frac{\partial}{\partial}\int_{0(\cdot)}\int_{(\cdot,\,)} (\ ,\ )\ ,\ ,\tag{B.7}$$

$$+\ \frac{\partial\ ^0}{\partial}(\cdot)\int_{(\cdot,\ ^0(\cdot))} (\ ,\ ^0(\cdot))\ ,\ \ \int_{0(\cdot)}\frac{\partial}{\partial}\left[\int_{(\cdot,\,)} (\ ,\ )\ ,\ \right]\ ,$$

$$+\ \int_{0(\cdot)} (\ (\cdot,\ ),\ )\frac{\partial\ (\cdot,\ )}{\partial}\ ,\ .$$

o o r o , r m 4o 4 (,) , m , $^0( )$ (,) o .

C. Derivation of main result 3

r 44or g o ( ), g (B.7), r 4 o m o 4 r o for ( ) m , . m g (2.2), o =(,) rg /or r m , rg m o 4 4o r o 4o r o of m ro or o o 2 2,

$$\frac{\lambda^2}{(\ )^2}\ \ \frac{\lambda^2}{(\ )^2}.\tag{.1}$$

4 ,

$$^0(\cdot) + \frac{\lambda}{\ }=,\tag{.2}$$

$$(\cdot,\ ) + \frac{\lambda\ }{[(\ )^2\ \ \lambda^2]^{1/2}},\tag{.3}$$

$$\frac{\partial}{\partial}\left(\ (\cdot,\ )\right) + \frac{\lambda(\ )^3}{\ }$$

o , 4 g for gr $_1(\cdot)$, $_2(\cdot)$ $_3(\cdot)$. B g g (.4), r o for r g (0, $_1(\cdot)$]

$$(\lambda^2)^{5/2}(^{1/2} \lambda) \leqslant (_1 \lambda^2)^{5/2}(_1^{1/2} \lambda). \quad (.7)$$

fo o

fo o

$$\left(1 \frac{\lambda^2}{}\right) \qquad \left(^{/2}\right.$$

r 4o > 0 $\lambda^2/$ > 0 o

$$\varphi( ) + \frac{[3 \rho(\lambda)]^2}{4\pi}(\lambda^2)\left[1 (\lambda^2/) (\lambda^2/)^{1/2}\right]$$

$$\geqslant \frac{[3 \rho(\lambda)]^2}{4\pi} . \qquad (17)$$

g of r $+ \frac{[3 \rho(\lambda)]^2}{4\pi}$ o o

$$_3( ) \leqslant 8\left(\frac{[3 \rho(\lambda)]^2}{4\pi}\right)^{3/2}\int_{\frac{[3 \rho(\lambda)]^2}{4\pi}\ _2( )/}^{} {}^{1/2} . \qquad (18)$$

rom (3), o r m of gr o 4o rg $\frac{[3 \rho(\lambda)]^2}{4\pi}\ _2( )/$ 1 gr (18) 4o rg o 4o . r for , $_3( )$ o m 4 g $^{3/2}$. o o $_2( )$ 4 g $(^{3/2})$ (o o o o r m, o r m 4 $^{3/2}$). o 4 of r o of $_1$ $_2$, g $\varphi( )$ r m m m $+ \lambda^2$ ($_1$, $_2$),

$$\varphi(\lambda^2)/ \leqslant \varphi( )/ \leqslant 1 \qquad (19)$$

for ($_1$, $_2$). g o (6), $_2( )$

$$( )\ ^1 \leqslant {}_2( ) \leqslant ( )\ ^{\varphi(\lambda^2)/}, \qquad (20)$$

r .

$$( ) \int_{_1( )}^{_2( )}\left(1 \frac{\lambda^2}{}\right)^{5/2}(^{1/2} \lambda) . \qquad (21)$$

g m o 4 ro m o for $_1( )$ $_2( )$ g (2) (3), gr (21) g of r m 4 g r [( ), ,1]' 4om) o o m o 4 r,

$$( ) \frac{2^4\pi^{3/2}}{3^4[\rho(\lambda)]^3}\ ^{3/2}. \qquad (22)$$

r mor , 4om $\varphi(\lambda^2)/$ 0 (20) o o m o 4

$$( )\ ^1 \leqslant {}_2( ) \leqslant ( ). \qquad (23)$$

4om (23) (13) (18) o o rg 4 g $( ) + \left(\frac{^{3/2}}{^3}\right)$.