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Does Evolution Solve the Hold-up Problem?

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# **Does Evolution Solve the Hold{up Problem?**

# 1 Introduction

pp pp rrl pb \_

pg pl pl r<sup>k</sup>r

ll r f r p p l r k r g p p g g m r b l p  
 p b m l ll r l p k r p p b f b r g p p g  
 g m b p q a l r m b b r k r p p b r p  
 b p j a p b k r g p p g m b b p r l  
 r p g p f r r b b k b f ll p g a p  
 • \W b f b r g p p g g m b m l l g m r f a l r r'  
 • \W b f m l b r l p p b p g m r f p'

l p p p r p g m l p l b m l r b l p  
p p l f r g m r f l r p f r m r g p p g  
r m p b l p b r r f g m r f  
p m p b l p l r m l r g p p g g m  
b p g g m r f a l r m p m l p l m m g m p  
b b b p r p m r r b r l m b r k p g  
r p r r l r p g m r f p b g m r b r k p g  
l m l f b l r l m b p r b l p r  
p m r b m g r b p b r n g r p r b l p p r ll  
b b l r l p p p l b p r b l m k r ll p  
p m p b l b m b ll l m p b r  
m p l W n g b p p m p l l  
l b g b r m l f r g p p g g m b r l  
g g b g p r l r p l m g b r f b p b r p p  
p p p p g m r f p b W l p p l  
k m p m l l p a b m p b  
b r b p b p p p g m r f p r p m b  
l b l l n g r f b p g p r l f r b  
r g p p g g m r r p r m p l p r  
p l f b b l r l m n g b r p p r r g p p g g m  
p m p p m p  
b l r p l l p r b r l p g p r p l  
p r p n g b p f r r p p b r p b b  
p r l r b r f b r l b p r p k  
b b r n g r p r l b ll p m r g  
f r ll b r r p l l r p b b p m p p r  
k p b r b p r r p p p f r l a l r m  
r b m m k b f r r p p r g m p b b r n g  
r p r b l r a r b r p b r p l b r b p  
b r n g r p r b l p r p b p a l r m b b m k  
b r l l f p m p r p k f r b p r p m



## 2 Investment and Bargaining

$\mathcal{W} = \{A, B, I\}$

$\Psi = \{I_0, I_1, \dots, I_N, I\}$

$V(I)$

$\mathcal{W} \parallel \mathcal{W}$

$\forall x (x \rightarrow y) \rightarrow (x \rightarrow y)$       $D_A \vdash \{V_1\} - x: V_1 \} \quad b, r, b$   
 $r \quad l, p, p \quad a \quad l, p \quad p, g, B \quad r, p, b \quad p, l, p, p$   
 $g \quad l, p, r \quad p, g \quad r, b \quad p, l, r \quad f \quad p, p \quad f, r, A \quad p$   
 $f \quad p, p, r, p \quad r, p \quad p, b \quad g, p \quad l, p, p \quad r, p, b$   
 $f \quad l, l \quad l, r, A \quad b \quad p \quad p, p, p, D \quad b \quad p, l \quad p, p$   
 $p, b \quad r, g, p, p, g, g, p \quad b \quad B, b \quad r, p, r \quad p, g, p$   
 $b \quad l, p, p, g, p \quad r, l, r, A \quad r, r, g \quad f, r, b, b, l, g, p$   
 $p \quad r \quad l, y, x \quad A \quad p, p \quad f \quad p, p, y, D \rightarrow D, r \quad b, r, b, p, r, l$   
 $p, p, r$   
 $f, r \quad r, p, p, g \quad b \quad l \quad p, r \quad p, l \quad l \quad p \quad r, b \quad g, p$   
 $r^f \quad b \quad a \quad l, r \quad b \quad p, b \quad p \quad p, p \quad p \quad f, l, l \quad b \quad b$   
 $p, p \quad g, p \quad a \quad b, r \quad W \quad p, l, l \quad f \quad g, p \quad r^f \quad a \quad l, r \quad p, r$   
 $l, r \quad b, r \quad r \quad g, p \quad r^f \quad a \quad l, r \quad p, p, g, r \quad p \quad p, p, p$   
 $k \quad p \quad p, l \quad A \quad l \quad b \quad r \quad g \quad l \quad l^* \quad l$



$\mathcal{S} \text{ gm } \mathcal{R}^f \text{ p m } r \text{ ll } p \text{ m } p \text{ b } b \text{ m } p \text{ gm}$   
 $p \text{ l } m \text{ l } p \text{ m } p \text{ b } l \text{ m } m \text{ gm } b \text{ p } l \text{ b}$   
 $p \text{ l } p \text{ r } \text{ ll } r \text{ p } b \text{ p } l \text{ b } r \text{ r } p \text{ f } l \text{ p } r$   
 $l \text{ p } \text{ ( } \text{ gm } \mathcal{R}^f \text{ p}$

### 3 Evolution

$\text{b } b \text{ r } g \text{ r } r \text{ p } l \text{ p } r \text{ p } r \text{ b } m \text{ b } l \text{ g } p \text{ f}$   
 $p \text{ f } b \text{ b } r \text{ g } p \text{ r } m \text{ r } r \text{ r}$   
 $b \text{ f } b \text{ l } p \text{ p } g \text{ m } p \text{ r } r \text{ p } p \text{ g}$   
 $l \text{ ) } p \text{ r } l \text{ b } p \text{ l } k \text{ ) } p \text{ p } g \text{ l } \text{ ) } b \text{ p } p$   
 $b \text{ r } f \text{ r } m \text{ r } k \text{ b } r \text{ 7}$   
 $r \text{ b } l \text{ r } r \text{ l } A \text{ p } B \text{ l } b \text{ r } \text{ population } \text{ f } N \text{ b}$   
 $r \text{ t} \in \{l; m\} \text{ r } l \text{ m } p \text{ p } \text{ f } g \text{ p } p \text{ l } p \text{ A } p$   
 $B \text{ p } p \text{ l } b \text{ p } m \text{ p } m \text{ r } g \text{ p } p \text{ g } g \text{ m } b \text{ f } r \text{ g } k$   
 $b \text{ m } g \text{ p } l \text{ b } l \text{ beliefs } b \text{ r } p \text{ p } \text{ f } p \text{ g}$   
 $p \text{ l } p \text{ r } r \text{ b } p \text{ r } g \text{ r } l \text{ r } b \text{ r } p \text{ ll } g \text{ p}$   
 $b \text{ r } l \text{ f } \text{ " } \text{ l } \text{ ) } p \text{ l } r \text{ A } l \text{ f } p \text{ r } p \text{ g } l \text{ r } B \text{ m } p$   
 $p \text{ l } \frac{3}{4} \text{ l } \text{ ) } p \text{ l } r \text{ B } l \text{ f } l \text{ r } A \text{ m } p \text{ b } \text{ "}$   
 $p \text{ } \frac{3}{4} \text{ r } r \text{ ) } l \text{ r } p \text{ p } b \text{ f } l \text{ m } p \text{ p } p \text{ b}$   
 $r \text{ p } p \text{ g } p \text{ b } p \text{ m } p \text{ l } \text{ f } r \text{ l } r \text{ p } g \text{ b}$   
 $l \text{ m } m \text{ g } m \text{ } \frac{3}{4} \text{ l } p \text{ p } l \text{ r } B \text{ m } p \text{ x}$   
 $\text{W } \text{ f } p \text{ l } \text{ [ } b \text{ p } l \text{ m } p$

**Assumption 1** (i) The pie division is small:  $V \text{ l } \text{ ) } > -$ . (ii) The population is large:  $V \text{ l } \text{ ) }^*$

♭           g♯           ♭                            μ ♭ r       p  
r       l       f       p       r       g       l       p       r       p       z μ  
l       f       p       r       g       l       p       r       p       adaptation  
r r p p r p p p       random mutation.       p       r       p       ♭       f l  
l       p g       r       r       ♭ g p ♭       p       ♭ p       f r       p ll       p g  
♭       l       f       p       r       g       ♭       ll       p       p g r       p       l       f       p g g p  
r       z μ       ♭       l       f       p       ♭       r       p       l       f       ll       p g  
p p       p r       ♭       p       μ r       p ♭ p g       p       ♭       r

$f: \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a continuous map. Let  $\mu \in \mathbb{R}^n$  and  $B(\mu, \epsilon)$  be a neighborhood of  $\mu$ . The basin of attraction of  $\mu$  is the set of points  $x \in \mathbb{R}^n$  such that  $f^k(x) \rightarrow \mu$  as  $k \rightarrow \infty$ . A single mutation is a point  $\mu' \in B(\mu, \epsilon)$ . A mutation connected set is a set of points  $\mu_1, \mu_2, \dots, \mu_{n-1}$  such that  $\mu_i \in B(\mu_{i-1}, \epsilon)$  for  $i = 2, \dots, n-1$ . The set  $M(\mu) \cap B(\mu, \epsilon)$  is the set of points  $\mu_1, \mu_2, \dots, \mu_{n-1}$  such that  $\mu_i \in B(\mu_{i-1}, \epsilon)$  for  $i = 2, \dots, n-1$ .

$\{x \in D_B \setminus I^* \mid V(I^*) - x \geq V(I) - I\}$

$$x^L \leftarrow \{x \in D_B \setminus I^* \mid V(I^*) - x \geq V(I) - I\}$$

$\{x \in D_B \setminus I^* \mid V(I^*) - x - I^* > V(I) - I\}$

$$x^M \leftarrow \{x \in D_B \setminus I^* \mid V(I^*) - x - I^* > V(I) - I\}$$

$x^M \leq x^L \leq x^M$

$$\begin{aligned}
 V^* - x^M &= N - I^* - V(I^*) - x^M - I^* - V(I^*) - x^M = N - N - I^* - I^* \\
 &> V(I^*) - x^M - I^* - x^M = N \\
 &\geq V(I^*) - x^M - I^* \\
 &\geq V(I) - I;
 \end{aligned}$$

$\{x \in D_B \setminus I^* \mid V(I^*) - x \geq V(I) - I\}$

**Proposition 3** *Let agents bargain according to the Nash demand game. The outcome  $x_0$  is locally stable if and only if  $x_0 \in \{I^*; V(I^*) - x; x\}$ , where  $x \leq x^L$ .*

$\{x \in D_B \setminus I^* \mid V(I^*) - x \geq V(I) - I\}$



$$\begin{array}{c}
\begin{array}{c}
p \ b \ b \ r \ X \rangle > r \ X \rangle \ b \ p \ r \ X > X^L \\
p \ r \ p \ p \ p \ p \ r \ p \ r \ b \ X^M > X^{NBS} \ p \ \rangle \ \parallel \ r \\
b \ p \ l \ \{ \ b \ b \ p \ p \ g \ p \ p \ p \ g \ l \ \} \ b \ r \ \{ \\
r \ X^L \rangle \geq r \ X^L - \rangle \ b \ p \ p \ p \ p \ r \ p \ r \ g \ p \ \}
\end{array} \\
\longrightarrow \longrightarrow \dots \longrightarrow X^L \longrightarrow X^L; \\
b \ l \ \{ \ r \ X^L \rangle \geq r \ X^L - \rangle \ b \ p \ p \ p \ p \ r \ p \ r \ g \ p \\
X^L \longrightarrow \longrightarrow \dots \longrightarrow X^L \longrightarrow
\end{array}$$

p p p b p b r p p p p p l l r p<sup>8</sup>  
l s p b r g p p g p r p g b r l f b  
l p p g p p /<sup>H</sup> b b V /<sup>H</sup> - /<sup>H</sup>

$\mathbb{P} \left( \sum_{i \in I} z_i \leq \sum_{i \in I} \bar{z}_i \right) \leq \mathbb{P} \left( \sum_{i \in I} z_i \leq \sum_{i \in I} \bar{z}_i \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right)$   
 $\leq \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right)$   
 $\leq \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right)$   
 $\leq \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right)$

## Appendix: Proofs

$\mathbb{P} \left( \sum_{i \in I} z_i \leq \sum_{i \in I} \bar{z}_i \right) \leq \mathbb{P} \left( \sum_{i \in I} z_i \leq \sum_{i \in I} \bar{z}_i \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right)$   
 $\leq \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right)$   
 $\leq \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right) = \mathbb{P} \left( \sum_{i \in I} (z_i - \bar{z}_i) \leq 0 \right)$

**Lemma 1** *Let  $z_1 < z_2 < \dots < z_n$  be demands in  $D \setminus I$  for some  $I \in \Psi$ . Assume that the set of demands following  $I$  for agents in the relevant population is  $\{z_i\}_i$*



$p$   $p$   $p$   $l$   $p$   $l$   $ll$   $g$   $p$   $p$   $b$   $r$   $l$   $p$   $b$   $p$   $k$   
 $l$   $b$   $p$   $p$   $V$   $l$   $z_i$   $p$   $p$   $p$   $g$   $p$   $l$   $ll$   $p$   $p$   $k$   
 $p$   $p$   $z_i$   $p$   $b$   $p$   $p$   $b$   $r$   $p$   $p$   $r$   $r$   
 $b$   $b$   $p$   $r$   $b$   $p$   $p$   $b$   $Q$   $p$   $r$   $pg$   $sp$   $M_i$   $p$   $N_i$   
 $p$   $r$   $l; y_i$   $r$   $p$   $p$   $x_i$   $b$   $r$   $l; y_i$   $p$   
 $f$   $ll$   $pg$   $l$   $sp$   $b$   $p$   $p$   $r$   $r$   $b$   $r$   $r$   $g$   $pg$   $l$   $p$   
 $ll$   $b$   $r$   $f$   $r$   $b$   $l$   $p$   $p$   $g$   $l$   $r$   $p$   $p$   $Q$   $pg$   $p$   $\square$   
 $r$   $b$   $p$   $r$   $r$   $r$   $p$   
 $\backslash W$   $r$   $f$   $f$   $r$   $p$   $p$   $b$   $Q$   $p$   $pg$   $p$   $sp$   $pg$   
 $p$   $b$   $pg$   $b$   $l$   $f$   $\%$



g f ll pg p p p p f l b b ll l b p l f ll pg  
r f pg) l; V l) - - ) p l p f p p p l b  
s ) p r p μ b % μ) - { l\*; y; x) } p x ≤ x<sup>L</sup>  
b b ) pg) p \* p p p l p b 1 p W μ<sub>1</sub> b  
% μ<sub>1</sub>) - % μ) r l i / l\* V l) - Ω - l ≤ V l) - r - l < Ω Ω f Ω  
V l\*) - x<sup>L</sup> N - l) = N - l\*) ≤ V l\*) - x) N - l) = N - l\*) p l g p  
p l p A x) μ<sub>j</sub>

$\mu$  is an equilibrium with outcome  $(I^*; y; x)$  with  $x \leq x^L$  to an equilibrium with outcome  $(I; V - \delta; -)$  is  $r x_{j-1} \mu \{r | r > N | - \frac{\hat{V} - \delta - \hat{I} + I^*}{V^* - x} \}$ .

$\parallel \mu \quad x^L \in \{x^M; x^M - -\}$ ;

**Lemma 5** *The number of mutations required to get from an equilibrium with outcome  $(I^*; y; x)$  with  $x \leq x^L$  to an equilibrium with outcome  $(I; V - \delta; -)$  is*

$$r x_{j-1} \mu \{r | r > N | - \frac{\hat{V} - \delta - \hat{I} + I^*}{V^* - x} \}. \quad (1)$$

$\mu$  is an equilibrium with outcome  $(I^*; y; x)$  and  $x < \mu \{x^M; x^{NBS}\}$ ;

**Lemma 6**

(i) *If  $\mu$  is an equilibrium with outcome  $(I^*; y; x)$  and  $x < \mu \{x^M; x^{NBS}\}$ ; the easiest transition away from  $\mu$  is  $\mu \{x^M; x^{NBS}\}$ .*

$r(x) \geq r(x)$ 
 $V^* - x^M \leq V - - - / \square$ 
 $x > x^L$ 
 $f(x^M) < x^{NBS}$

$f(x) - x^M > x^{NBS}$

$I^*; y; x$ 
 $y \leq V^* - x$ 
 $I^*; y; x$

**Lemma 7** From an outcome  $I^*; y; x$  the easiest transition in which investment is at all times efficient, but which ends with different demands, is to an outcome  $I^*; y'; x'$  where  $x' \leq x$ ;  $x' \geq x$ ; or  $V^* - -$ .

$f(x)$ 
 $r(x)$ 
 $r(x)$

**Lemma 8**

- (i) Moving from  $x$  to  $x - -$  takes  $N \ln \frac{-\delta}{x}$  mutations to pop A.
- (ii) Moving from  $x$  to  $x - -$  takes  $N \ln \frac{V^* - \delta}{V^* - x}$  mutations to pop B.
- (iii) Moving from  $x$  to  $-$  takes  $N \ln \frac{V^* - \delta}{x}$  mutations to pop B.
- (iv) Moving from  $x$  to  $V^* - -$  takes  $N \ln \frac{V^* - \delta}{x}$  mutations to pop A.

$f(x)$ 
 $r(x)$ 
 $r(x)$ 
 $N \square$

**Lemma 9**

- (i) If  $- < x < V^* - -$ , then moving from  $x$  to  $x - -$  takes fewer mutations than moving from  $x$  to  $-$ , and moving from  $x$  to  $x - -$  takes fewer mutations than moving from  $x$  to  $V^* - -$ .

(ii) If  $x_{1-}$  - then moving from  $x$  to

$x^{NBS} < x \leq x^L$   $\mu; \mu'_{\neq \delta}$   $x < x^{NBS}$   $\mu; \mu'_{\neq \delta}$   $x < x^{NBS}$   
 $x^{NBS} < x \leq x^L$   $\mu; \mu'_{\neq \delta}$   $x < x^{NBS}$   $\mu; \mu'_{\neq \delta}$   $x < x^{NBS}$

A series of mathematical expressions and symbols including  $\mu; \mu'_{\neq \delta}$ ,  $x < x^{NBS}$ ,  $x^{NBS} < x \leq x^L$ , and various Greek letters and subscripts.

**Lemma 11** *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome,  $\{I^H; V^H - x^{\max} | I^H; x^{\max} | I^H\}$ , is a subset of the unique locally stable set.*

$\mu \in T \mu^H$   $\mu^H \in T \mu^H$   $\{I^H; V^H - x^{\max} | I^H; x^{\max} | I^H\}$   
 $\mu \in T \mu^H$   $\mu^H \in T \mu^H$   $\{I^H; V^H - x^{\max} | I^H; x^{\max} | I^H\}$   
 $\mu \in T \mu^H$   $\mu^H \in T \mu^H$   $\{I^H; V^H - x^{\max} | I^H; x^{\max} | I^H\}$

**Lemma 12** *Let surplus be divided by the ultimatum game. Agents in population A receive a payoff of at least  $V^H - I^H - x^{\max} | I^H$  in every equilibrium.*

$\mu \in T \mu^H$   $\mu^H \in T \mu^H$   $\{I^H; V^H - x^{\max} | I^H; x^{\max} | I^H\}$

**Lemma 13** *Let surplus be divided by the 'ultimatum' game. If  $V | I - I - x \geq V^H - I^H - x^{\max} | I^H$ , then there exists an equilibrium  $\mu$  such that  $\mu \in \Theta^L$  and  $\mu \in T \mu^H$ .*

r <sup>f</sup> m m <sup>f</sup> r m m m // p □  
 b <sup>f</sup> r <sup>f</sup> m g b m A b g r b p  
 b g p b b l a l r m b p b r r g l r p b b b  
 m r r p b l m m r m p b r b p  
 g l r p r r p b l r b f b r l m m  
 r <sup>f</sup> f r p r m m m // l p l k b b b  
 p g l // l b b s m l p l b p m g l b  
 b // l □



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Y L E Y Econometrica

Y L E Y Journal of Economic Theory

Y L E Y Review of Economic Studies