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Does Evolution Solve the Hold-up Problem?

Tore Ellingsen
Department of Economics, Stockholm School of Economics
Stockholm, Sweden

Jack Robles
Department of Economics, University of Colorado at Boulder
Boulder, Colorado

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Center for Economic Analysis
Department of Economics

University of Colorado at Boulder
Boulder, Colorado 80309

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1 Introduction

[illegible]

2 Investment and Bargaining

[illegible]

[illegible] $D_A \vdash \{V\ I() - x; V\ I()\}$ [illegible]

[illegible] B [illegible] A [illegible]

[illegible] A [illegible] D [illegible]

[illegible] B [illegible] A [illegible]

[illegible] $I; y\ x()$ [illegible] A [illegible] $y,\ D \to D$ [illegible]

[illegible]

[illegible] A [illegible] $I \vdash I^*$ I

3 Evolution

A *B* *population* *N* $t \in \{1; \ldots\}$ *A* *B* *beliefs* *A* *B* *B* *A* *I* *B* *x*

Assumption 1 *(i) The pie division is small: V I > –. (ii) The population is large: V I** *adaptation*

random mutation.

Θ

basin of attraction

μ *B* μ μ'

μ' μ μ' *single mutation neighborhood* μ $\mu' \in M$ μ μ' μ

X *mutation connected set*

μ_1 $\mu_n \subset X$ $\mu_2, \ldots, \mu_{n-1}$

k $; \ldots; n;$ *M* μ $\cap$ *BX* μ *n*

$$x^L = \max\{x \in D_B(I^*) \mid (V(I^*) - x)\frac{N-1}{N} - I^* \geq V(I) - I\}.$$

$$x^M = \max\{x \in D_B(I^*) \mid V(I^*) - x - I^* > V(I) - I\}.$$

$$x^M \leq x^L \leq x^M.$$

$$
\begin{aligned}
(V^* - x^M)(N-1)/N - I^* &= V(I^*) - x^M - I^* - (V(I^*) - x^M)/N \\
&\quad - (N-1)/N \\
&> V(I^*) - x^M - I^* - x^M/N \\
&\geq V(I^*) - x^M - I^* \\
&\geq V(I) - I;
\end{aligned}
$$

$(N-1)/N$ x^L B A

Proposition 3 *Let agents bargain according to the Nash demand game. The outcome ‰ is locally stable if and only if ‰ = {I*; V(I*)−x; x}, where $x \leq x^L$.*

… Θ … $r(\mu, \mu')$ … μ … μ' … $\mathcal{G}$ … *resistance* … *cost* … $\mu \rightarrow \mu'$ … $r(\mu, \mu')$ … μ-*tree*, Γ … $\mathcal{G}$ … $\mu' \neq \mu$ … μ … Γ … *stochastic potential* … μ … μ …

Theorem 1 *An equilibrium* μ *is stochastically stable if and only if no other equilibrium has lower stochastic potential.*

… $I = I^*$ … $y = V(I^*) - x$ … B … x … $x < x^{NBS}$ … $V(I^*) = \ldots$ … x … $x > x^{NBS}$ … x … $r(x)$ … B … A … $(I, V(I))$ … $(I^*, V(I^*))$ … $r(x)$ … $(I^*, V(I^*) - x, x)$ … $(I, V(I))$ …

$r\ x() > r\ x()$ $x > x^L$

$x^M > x^{NBS}$

$r\ x^L() \geq r\ x^L - -;$

$$- \longrightarrow\ - \longrightarrow ::: \longrightarrow x^L - - \longrightarrow x^L;$$

$r\ x^L() \geq r\ x^L - -()$

$$x^L \longrightarrow - \longrightarrow\ - \longrightarrow ::: \longrightarrow x^L - -:$$

[illegible]

Appendix: Proofs

[illegible]

Lemma 1 *Let* $z_1 < z_2 ::: < z$ *be demands in* $D\ I()$ *for some* $I \in \Psi$*. Assume that the set of demands following* I *for agents in the relevant population is* $\{z_l\}_l$

B [illegible] $I;y$ [illegible] A [illegible] $I;y$ [illegible] μ′ [illegible] $\square$

Lemma 4 *Let* μ′ *(‰ μ′ { I′; y′; x′ }) be an equilibrium. If I ∠ I′ and $y - I \geq y' - I'$, then the population can get from* μ′ *to an equilibrium* μ *with* ‰ μ { $I; y; x$ } *through a sequence of single mutation transitions.*

[illegible] μ′ [illegible] B [illegible] A [illegible] I [illegible] y [illegible] j [illegible] B x I x $V(I) - y$ [illegible] A [illegible] $I;y$ [illegible] A [illegible] x I x [illegible] j [illegible] B [illegible] $y - I > y' - I'$ [illegible] $I;y$ [illegible] μ [illegible] ‰ μ { $I;x;y$ } [illegible] $y - I$ $y' - I'$ [illegible] μ_1 [illegible] ‰ μ_1 { $I;y;x$; $I';y';x'$ } [illegible] μ [illegible] ‰ μ { $I;y;x$ } $\square$

[illegible] S [illegible] μ [illegible] ‰ μ [illegible] ‰ μ { $I; V(I) - x; x$ } [illegible] $I \angle I^*$ [illegible] $V(I^*) - - - I^* > V(I) - x - I;$ [illegible] μ^L [illegible] ‰ μ^L { $I^*; V^* - -; -$ } [illegible] I I^* [illegible] $x > x^L$ [illegible] $V(I) - - - I \geq V(I) - x - I$ [illegible] μ [illegible] ‰ μ { $I; V(I) - -; -$ } [illegible] μ^L [illegible] ‰ μ^L { $I^*; V^* - -; -$ } [illegible] I I^* $x > x^L$ [illegible] $V(I) - - - I < V(I) - x - I$

$I; V \ I$ $\{ I^*; y; x \}$ $x \leq x^L$

$V \ I^* - x^L \ N - I = N - I^* \leq V \ I^* - x \ N - I = N - I^*$

A ... $I; V - -; -)$ ... B ...

$x^L \in \{x^M; x^M - -\}$:

Lemma 5 *The number of mutations required to get from an equilibrium with outcome $(I^*; y; x)$ with $x \leq x^L$ to an equilibrium with outcome $(I; V - -; -)$ is $r(x) = \min\{r | r > N(1 - \frac{\hat{V} - \delta - \hat{I} + I^*}{V^* -})\}$.*

r ... B ...

$$\frac{N - r}{N}(V^* - x) - I^* < V - I - -; \quad (\ )$$

□

Lemma 6

(i) If µ is an equilibrium with outcome $(I^*; y; x)$ and $x < \min\{x^M; x^{NBS}\}$; the easiest transition away from ‰ µ) Sqlx3655129Tf(0.82TΩ0.9482 2 2 2F7283m)-343(with)-38326

$x > x^L$ ... $x \ldots x^L \ldots x^M$ ... $x^M < x^{NBS}$; ... $r(x) \geq r(x)$ ... $V^* - x^M - - \leq V - - - I$ □

... $x \ldots x^M > x^{NBS}$ ... $(I; V - -; -)$ ... $(I^*; y; x - -)$ ...

... $(I^*; y; x)$ ... $y \ldots V^* - x$ ... $(I^*; y'; x')$ ...

Lemma 7 *From an outcome $(I^*; y; x)$ the easiest transition in which investment is at all times e–cient, but which ends with diﬁerent demands, is to an outcome $(I^*; y'; x')$ where $x' \ldots x - -; x \ldots -; -;$ or $V^* - -$.*

□

Lemma 8

(i) Moving from x to $x - -$ takes $N(\ldots - \frac{-\delta}{\ldots})$ mutations to pop A.

(ii) Moving from x to $x \ldots -$ takes $N(\ldots - \frac{V^* - \ldots - \delta}{V^* - \ldots})$ mutations to pop B.

(iii) Moving from x to $-$ takes $N(\frac{V^* - \ldots}{V^* - \delta})$ mutations to pop B.

(iv) Moving from x to $V^* - -$ takes $N(\frac{\ldots}{V^* - \delta})$ mutations to pop A.

... N □

Lemma 9

(i) If $- < x < V^* - -$, then moving from x to $x - -$ takes fewer mutations than moving from x to $-$, and moving from x to $x \ldots -$ takes fewer mutations than moving from x to $V^* - -$.

(ii) If x $_{\lfloor}$ *– then moving from x to*

[illegible] $x^L > x^{NBS}$ [illegible] Θ^1 [illegible] Γ^1 [illegible] $x < x^{NBS}$ [illegible] $x^{NBS} < x \leq x^L$ [illegible] $x > x^{NBS}$ □

[illegible]

Lemma 11 *Let surplus be divided by the ultimatum game. The component with the subgame perfect outcome, $(l^H, V^H - x^{\max}(l^H), x^{\max}(l^H))$, is a subset of the unique locally stable set.*

[illegible]

□

Lemma 12 *Let surplus be divided by the ultimatum game. Agents in population A receive a payoff of at least $V^H - l^H - x^{\max}(l^H)$ in every equilibrium.*

[illegible] □

Lemma 13 *Let surplus be divided by the 'ultimatum' game. If $V(l) - l - x \geq V^H - l^H - x^{\max}(l^H)$, then there exists an equilibrium μ such that $\mu \in \Theta^L$ and ‰$(\mu) = (l, V(l) - x, x)$.*

6 References

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