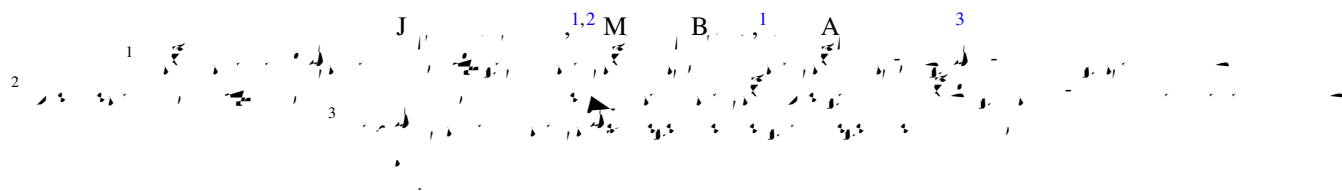
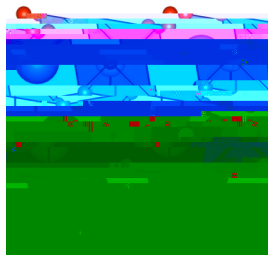


## Origins versus fingerprints of the Jahn-Teller effect in $d$ -electron $ABX_3$ perovskites





Q



$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1$  with  $\mathcal{H}_0 = \sum_{i=1}^3 \frac{p_i^2}{2m_i} + \sum_{i=1}^3 \frac{1}{2} m_i \omega_i^2 x_i^2$  and  $\mathcal{H}_1 = \sum_{i=1}^3 \lambda_i x_i^3$  ( $\lambda_i = 0, F$ )  
 where  $\mathcal{H}_0$  is the harmonic oscillator Hamiltonian and  $\mathcal{H}_1$  is the cubic anharmonicity.

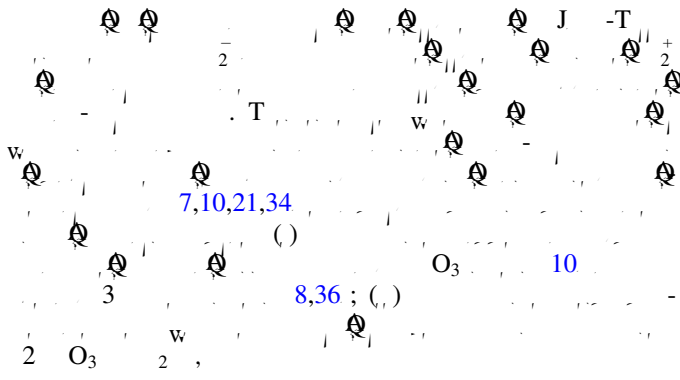


TABLE I. DFT calculations of the crystal field splitting energy  $E_{CF}$  (in eV) for the  $d^6$  configuration of  $\text{Fe}^{2+}$  in the octahedral  $\text{O}_h$  symmetry. The calculations were performed using the Perdew-Burke-Ernzerhof (PBE) functional with the spin-orbit coupling (SOC) included. The results are shown for the ground state (GS) and the excited states (ES) of the  $d^6$  configuration. The observed values (OBS) are also shown for comparison. The error bars are given in parentheses. The AFM and AFMG denote the antiferromagnetic and antiferromagnetic ground state, respectively. The AFMA denotes the antiferromagnetic magnetic anisotropy. The AFMG denotes the antiferromagnetic magnetic ground state. The AFMA denotes the antiferromagnetic magnetic anisotropy. The AFMG denotes the antiferromagnetic magnetic ground state.

$E_i$	$(\bar{2}, \bar{2}, 2)$	$(\bar{2}, \bar{2}, 2)$ (GGA + SOC)		$(\bar{2}, \bar{2}, 2)$ (HSE06)	
		$\Delta_{\text{OBS}}$	$\Delta_{\text{OBS}} (\text{eV})$	$\Delta_{\text{OBS}}$	$\Delta_{\text{OBS}} (\text{eV})$
L T $\text{O}_3$	$1 (\frac{1}{2})$	0	(0)	0	(0)
L M $\text{O}_3$	$4 (\frac{3}{2}, \frac{1}{2})$	0	(0)	0	(0)
L $\text{O}_3$	$2 (\frac{2}{2})$	-297	(-237)	0.42	(-428)
KF $\text{F}_3$	$6 (\frac{3}{2}, \frac{2}{2}, \frac{1}{2})$	-			

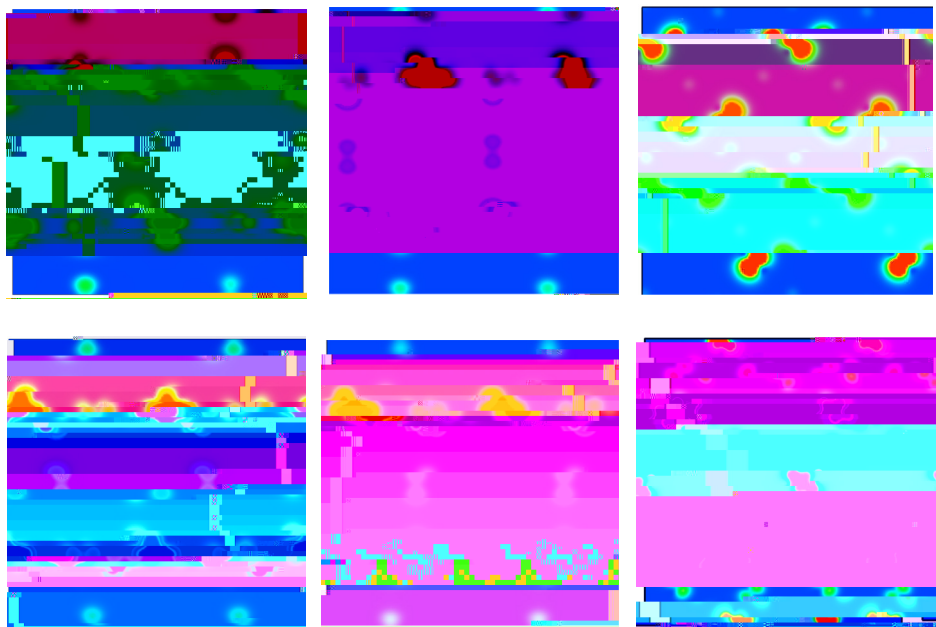




TABLE II. A summary of the  $Q_2^+$  motion in  $\text{LaTiO}_3$ ,  $\text{LaMnO}_3$ , and  $\text{LaFeO}_3$ . The table lists the  $Q_2^+$  wave vector  $Q$ , the magnetic structure (M), the spin configuration (S), and the  $Q_2^+$  displacement  $\delta$  (in Å) for the  $E_g$  and  $T_{2g}$  modes. The values in parentheses are the  $Q_2^+$  displacement  $\delta$  (in Å) for the  $E_g$  and  $T_{2g}$  modes, respectively. The values in blue are the  $Q_2^+$  displacement  $\delta$  (in Å) for the  $E_g$  and  $T_{2g}$  modes, respectively.

	$Q$	M	S	$\frac{1}{2} \begin{pmatrix} + \\ 3 \end{pmatrix} C_4 Q_2^+$ (E <sub>g</sub> )	$\frac{1}{2} \begin{pmatrix} - \\ 3 \end{pmatrix} C_4 Q_2^+$ (E <sub>g</sub> )
LTiO <sub>3</sub>	0.93	AFMG		0.040 (0.041 56)	
LMnO <sub>3</sub>	0.94	AFMA		0.324 (0.357 30)	
LFeO <sub>3</sub>	0.95	AFMC		0.005 (0.009 57)	0.093 (0.079 57)
				0.078 (0.090 58)	
KFF <sub>3</sub>	1.00	AFMG			0.104 ( )
KCF <sub>3</sub>	1.01	AFMG			0.003 ( )
KCF <sub>3</sub>	0.99	AFMA			0.336 (0.316 28)
					0.300 (0.299 28)
					0.335 (0.355 29)

C. The  $Q_2^+$  motion in  $\text{LaTiO}_3$  and  $\text{LaMnO}_3$  is a consequence





AFMG AFMC : ( )  
T  
S III C  
10  
O J -T A H)  
31,55  
J -T

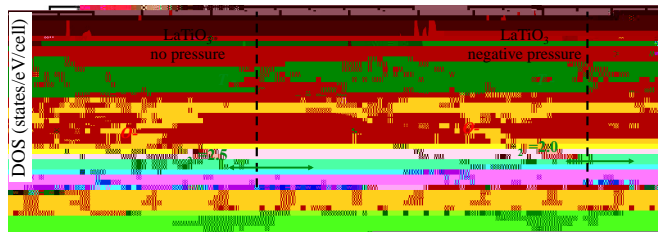
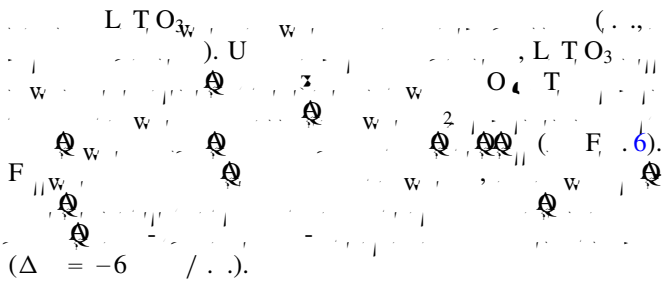


FIG. 6. P  $Q_2^-$   $O_4$   $(\bar{2}, \bar{2}, 2)$   $T_2$   $(\bar{2}, \bar{2}, 2)$ ,  $Q_2^-$   $L T_2 O_3$   $(\bar{2}, \bar{2}, 2)$   $w$   $(\bar{2}, \bar{2}, 2)$ .



APPENDIX C: ENERGY GAIN ASSOCIATED WITH  $Q_2^+$  AND  $Q_2^-$  OCTAHEDRAL DEFORMATION MODE

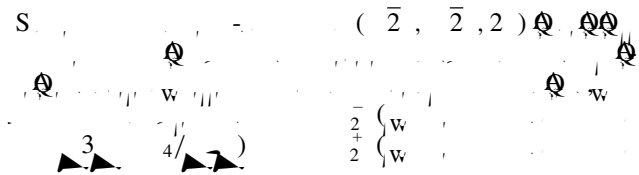
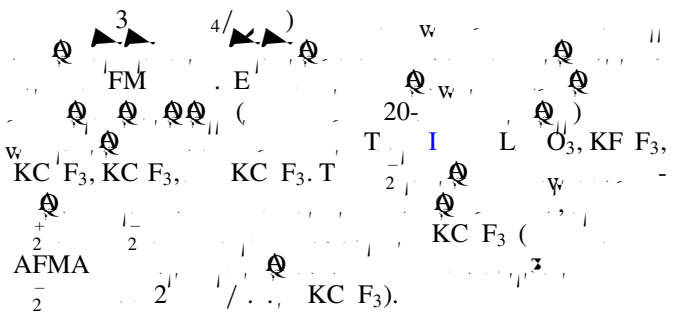


TABLE I. Energy levels  $E$  (in eV) for various states and magnetic orders.

State	$\Delta_{2+} (eV)$	$\Delta_{2-} (eV)$
KF $F_3$	-2739	-2760
KC $F_3$	-2976	-3004
L $O_3$	-1153	-1325
KC $F_3$	-1295	-1298
KC $F_3$	-1019	-1020



APPENDIX D: POTENTIAL ENERGY SURFACES ASSOCIATED WITH  $Q_2^-$

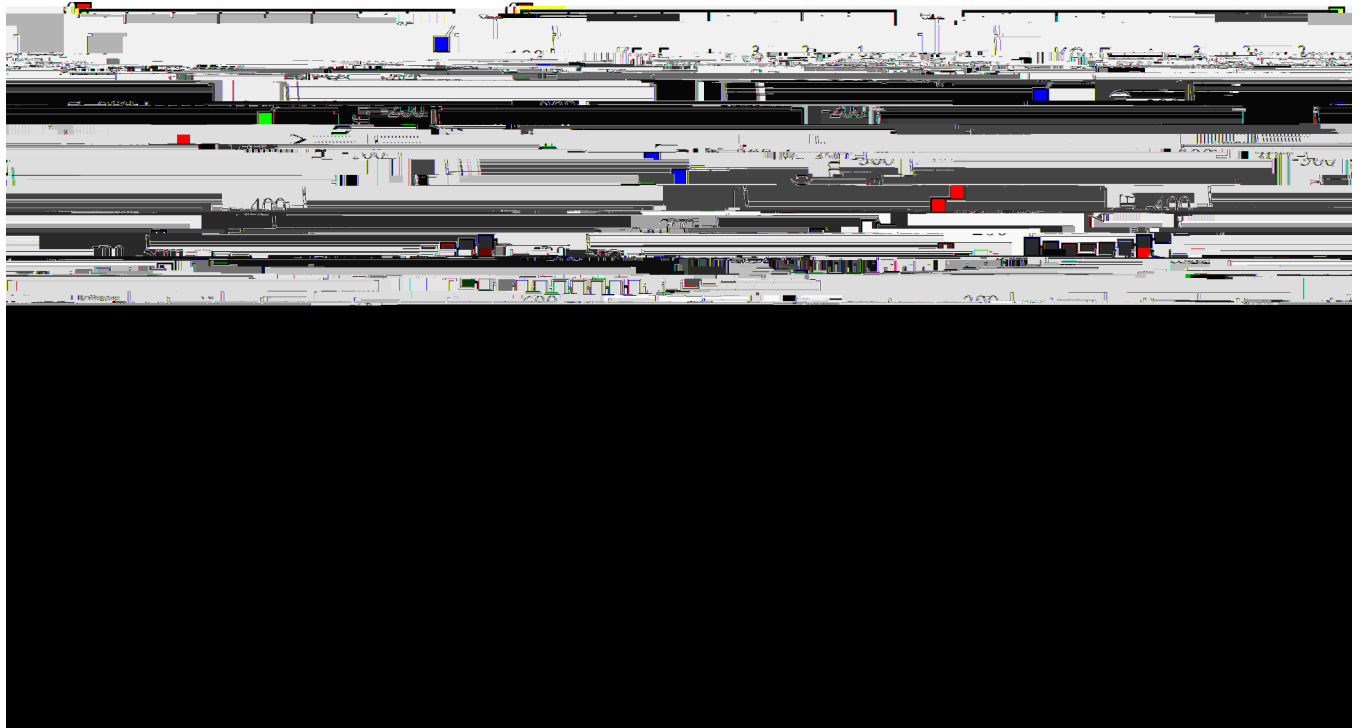
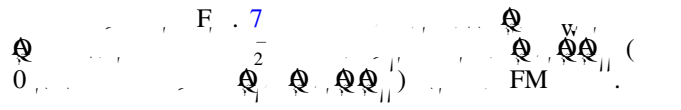


FIG. 7. P  $Q_2^-$   $O_4$   $(\bar{2}, \bar{2}, 2)$   $T_2$   $(\bar{2}, \bar{2}, 2)$ ,  $Q_2^-$   $L T_2 O_3$   $(\bar{2}, \bar{2}, 2)$   $w$   $(\bar{2}, \bar{2}, 2)$   $F$   $(\bar{2}, \bar{2}, 2)$   $AFMA$   $(\bar{2}, \bar{2}, 2)$   $FM$   $(\bar{2}, \bar{2}, 2)$   $P$   $(\bar{2}, \bar{2}, 2)$ .



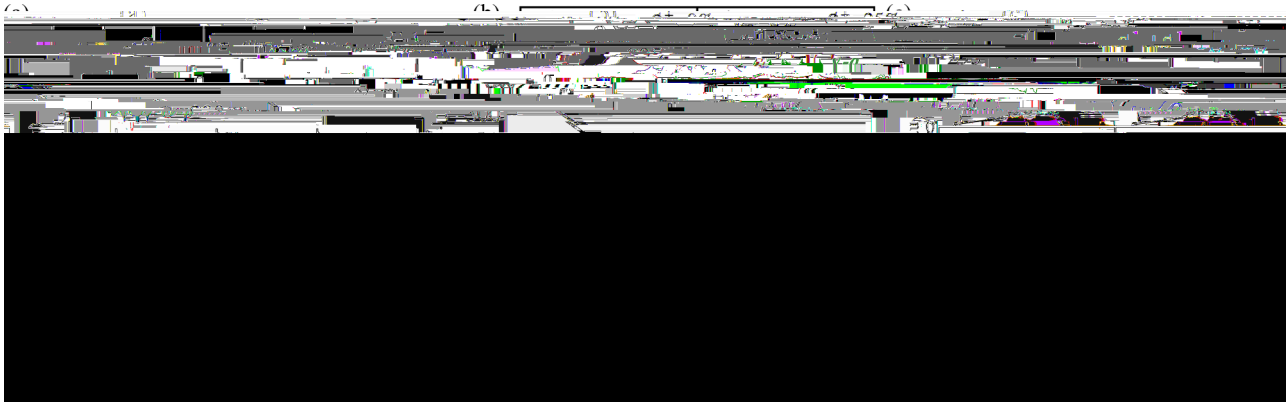
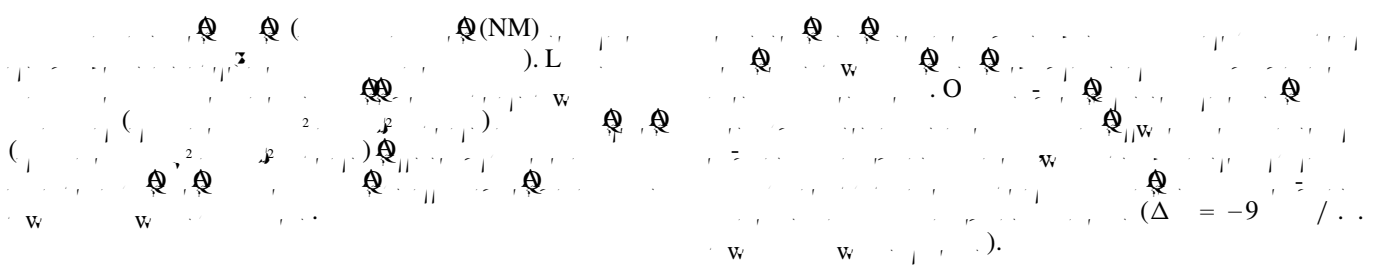
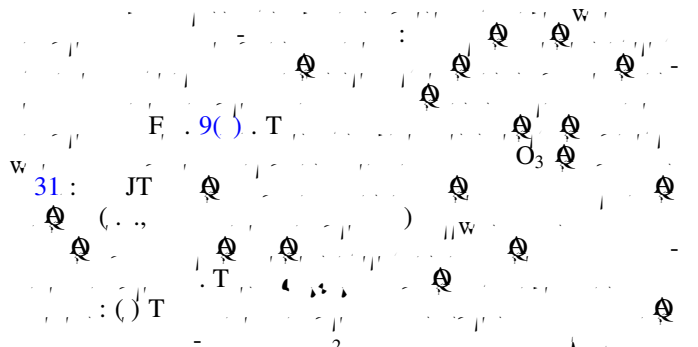
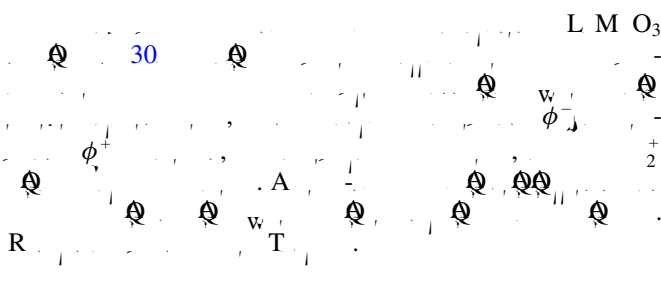


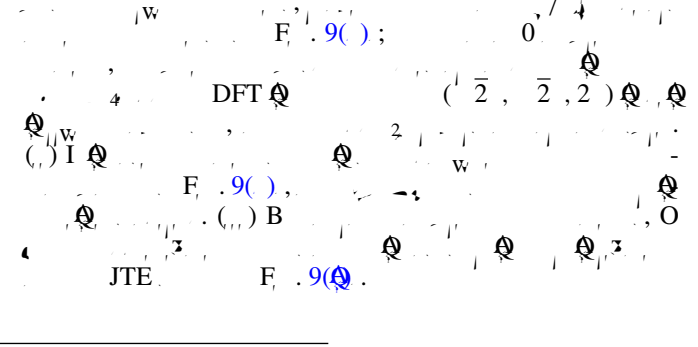
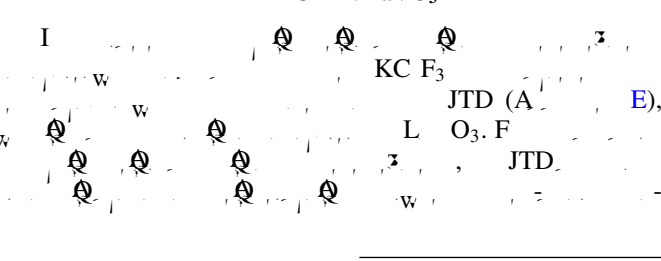
FIG. 9. Crystal structure of  $\text{LaMnO}_3$  showing the Jahn-Teller distortion. The structure is shown in the  $ab$ -plane, with the  $c$ -axis perpendicular to the page. The Mn sites are shown as blue spheres, and the O sites as red spheres. The Jahn-Teller distortion is indicated by the displacement of the Mn sites from their high-symmetry positions. The structure is shown in the  $ab$ -plane, with the  $c$ -axis perpendicular to the page. The Mn sites are shown as blue spheres, and the O sites as red spheres. The Jahn-Teller distortion is indicated by the displacement of the Mn sites from their high-symmetry positions.



APPENDIX G: SYMMETRY MODE ANALYSIS OF  $\text{LaMnO}_3$  EXPERIMENTAL STRUCTURES



APPENDIX H: COOPERATING AND COMPETING OCTAHEDRAL ROTATIONS AND JAHN-TELLER EFFECT IN  $\text{LaVO}_3$



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