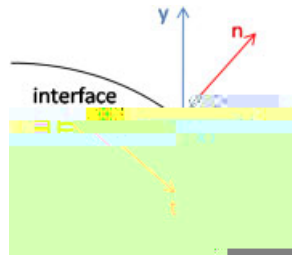


FIGURE 1. Geometric domain  $\Omega$  and its sub-domains.







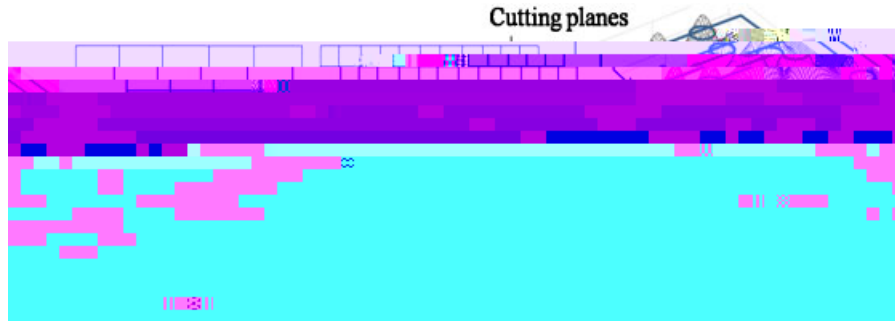


FIGURE 3. (a)  $E_{\text{fac}}(d) = \dots$ ; (b)  $E_{\text{fac}}(d) = \dots$ ; (c)  $E_{\text{fac}}(d) = \dots$

$$N_I(\cdot) = \begin{cases} 0 & \text{if } \dots \\ 0 & \text{if } \dots \end{cases} \quad (17)$$

...  $N_I(\cdot)$  ...  $H(\cdot)$  ...  $\chi(\cdot)$  ...  $\phi(\cdot)$  ...  $\nabla\phi(x)$  ...

$$(\cdot) = \frac{\nabla\phi(x)}{\|\nabla\phi(x)\|} \quad (18)$$

...  $H(\phi)$  ...  $\chi_j(\cdot) = |\phi(\cdot)| - |\phi(\cdot_j)|$  ...

$$H(\phi) = \begin{cases} 1 & \text{if } \dots \\ 0 & \text{if } \dots \end{cases} \quad \text{and } \chi_j(\cdot) = |\phi(\cdot)| - |\phi(\cdot_j)| \quad (19)$$

...  $\chi_j(\cdot)$  ...  $\chi_a$  ...  $\chi_c$  ...  $\chi_e$  ...  $\chi_f$  ...

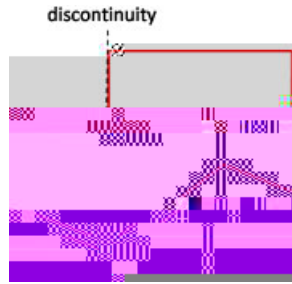


FIGURE 4. A schematic of a layered material with a surface discontinuity. (a) shows a layered material with a surface discontinuity. (b) shows a layered material with a surface discontinuity and a surface defect.

*Bulk energy*

So the bulk energy  $\tilde{W}_b^e$  is defined as the energy of the bulk material,  $\tilde{W}_b^e = \int_{\Omega} \tilde{w}_b^e(\mathbf{e}) d\Omega$ , where  $\tilde{w}_b^e$  is the bulk energy density,  $\mathbf{e}$  is the strain tensor, and  $\Omega$  is the volume of the bulk material.

$$\delta \tilde{W}_b^e = \int_{\Omega} \delta \mathbf{e}^e : \mathbf{e}^e d\Omega = \delta \mathbf{e}^T \cdot \int_{\Omega} \mathbf{e}^T \{ \mathbf{e} \}^e d\Omega \quad (20)$$

where  $\mathbf{e}^e$  is the strain tensor,  $\mathbf{e}^T$  is the transpose of the strain tensor, and  $\{ \mathbf{e} \}^e$  is the strain tensor with a surface discontinuity. The energy of the bulk material is  $\tilde{W}_b^e = \int_{\Omega} \tilde{w}_b^e(\mathbf{e}) d\Omega$ , where  $\tilde{w}_b^e$  is the bulk energy density,  $\mathbf{e}$  is the strain tensor, and  $\Omega$  is the volume of the bulk material.





*External energy*

External energy functional  $\tilde{W}_e$  is defined as follows:

$$\delta \tilde{W}_e = \delta \int_{\Omega} e^T d\Omega + \delta \int_{\partial\Omega_F} e^T d\Gamma \quad (30)$$

*Final XFEM equation*

Using the variational principle (20), (22), (28), and (30), the final XFEM equation is:

$$\left( \frac{e}{b} + \frac{e}{d} + \frac{e}{s} \right) \cdot e = \frac{e}{b} - \frac{e}{s} \quad (31)$$

where  $e$  is the strain tensor,  $b$ ,  $d$ , and  $s$  are material parameters, and  $\nabla$  is the gradient operator.

$$e = [ \nabla \cdot u ] \in \mathbb{R}^{d \times d}$$

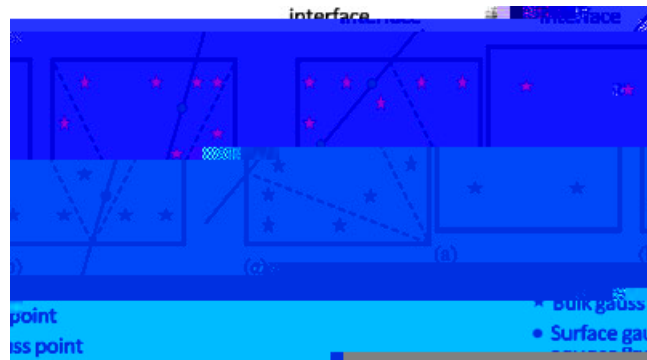


FIGURE 5. The case  $\alpha = 1$ : (a)  $\phi = a$ ; (b) and (c)  $\phi = c$ .

Consider the case  $\alpha = 1$ . The bulk Gauss points are  $\mathbf{x}_i$  and  $\mathbf{x}_j$  in  $\Omega_1$ , and  $\mathbf{x}_k$  and  $\mathbf{x}_l$  in  $\Omega_2$ . The surface Gauss points are  $\mathbf{x}_m$  and  $\mathbf{x}_n$  on the interface. The bulk Gauss points are  $\mathbf{x}_i$  and  $\mathbf{x}_j$  in  $\Omega_1$ , and  $\mathbf{x}_k$  and  $\mathbf{x}_l$  in  $\Omega_2$ . The surface Gauss points are  $\mathbf{x}_m$  and  $\mathbf{x}_n$  on the interface. The bulk Gauss points are  $\mathbf{x}_i$  and  $\mathbf{x}_j$  in  $\Omega_1$ , and  $\mathbf{x}_k$  and  $\mathbf{x}_l$  in  $\Omega_2$ . The surface Gauss points are  $\mathbf{x}_m$  and  $\mathbf{x}_n$  on the interface.



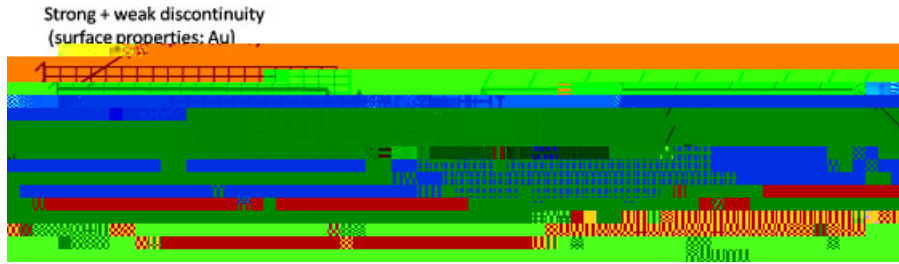


FIGURE 6. Schematic of the layered material system used for XFEM analysis.

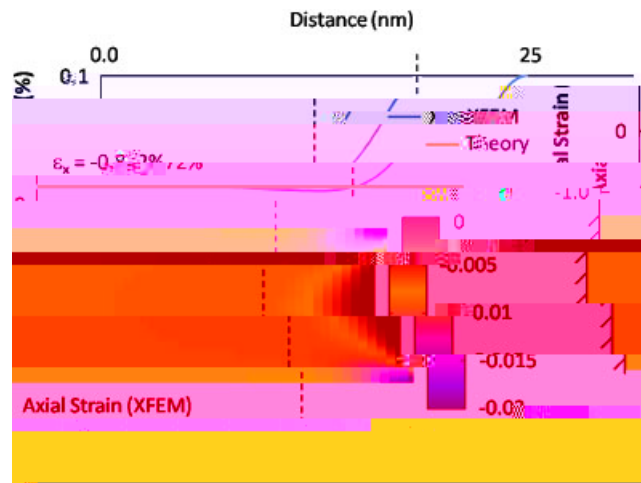


FIGURE 7. Axial strain distribution along the crack plane for the layered material system. (40)

Figure 7 shows the axial strain distribution along the crack plane for the layered material system. The axial strain is compared with the theoretical strain (40) and the XFEM results. The theoretical strain is zero, while the XFEM results show a significant strain concentration near the crack tip. The maximum strain concentration is approximately -0.02% at the crack tip. The strain distribution is shown in Figure 7, where the theoretical strain is zero and the XFEM results show a significant strain concentration near the crack tip.





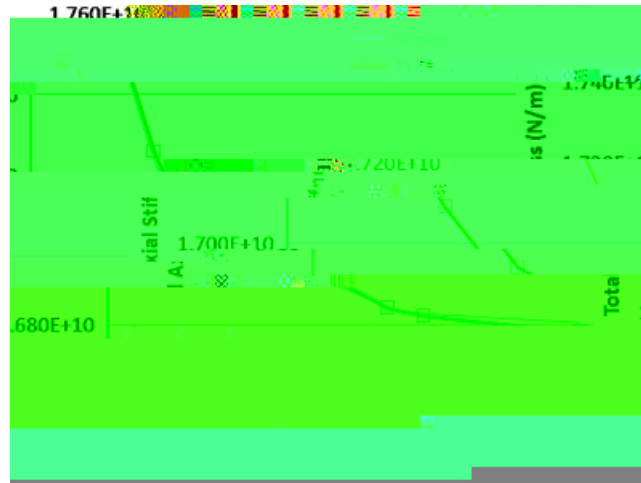


Fig. 9. Stress distribution in the material.

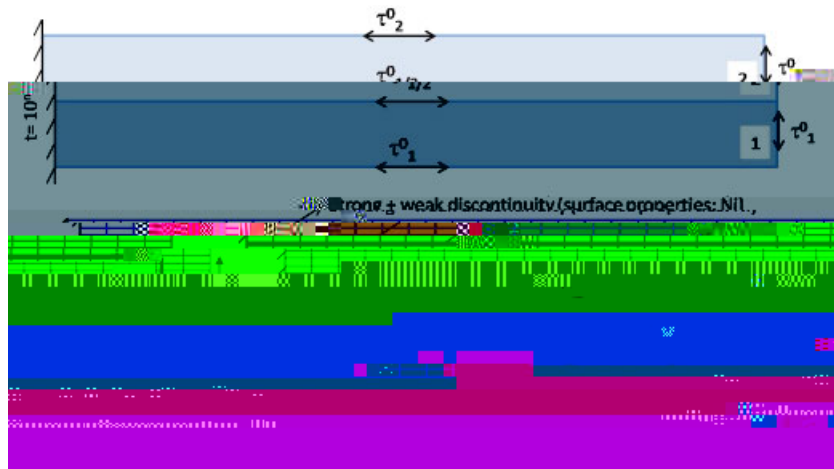


Fig. 10. Stress distribution in the material.

If  $\tau_1^0, \tau_2^0$ , and  $\tau_{12}^0$  are the stress components (in the  $x$ - $y$  plane) at the interface  $x = 0$ , then the stress components in the  $x$ - $y$  plane at the interface  $x = t$  are given by:

$$M = (\tau_{12}^0)(t-x) - (\tau_2^0)x + (\tau_1^0) \frac{t}{2} - x \quad (47)$$





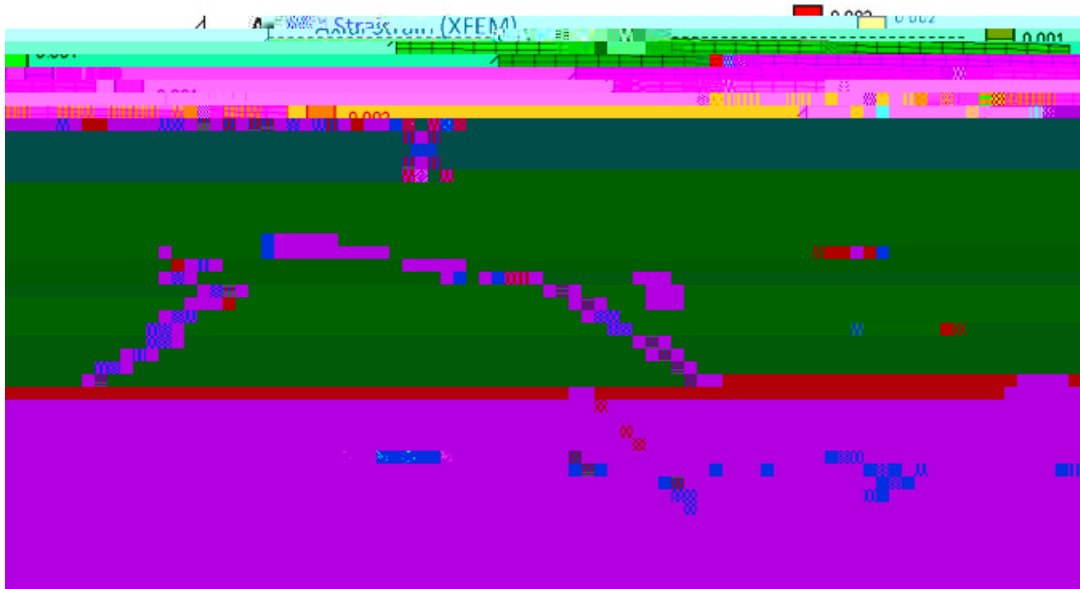


Figure 11. (a) A finite element analysis (XFEM) plot showing stress distribution in a layered material. The plot is titled "Strain (XFEM)". It features a color-coded stress field with a legend at the top right showing values from 0.000 to 0.002. The material is divided into several horizontal layers of different colors (green, blue, red, purple). A crack is visible, propagating through the layers. The stress concentration is highest at the crack tip.

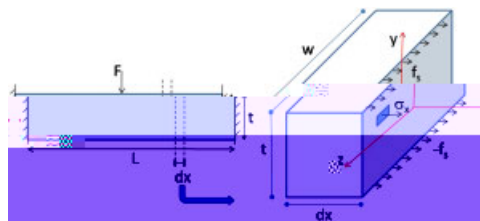


Figure 12. Schematic diagram of a beam element showing forces and dimensions.

The stress distribution in the beam element is given by the following equation:

$$\int_A \sigma_x y dA + 2 \int_0^t f_s \frac{t}{2} dz = -M \quad (53)$$

where  $\sigma_x$  is the normal stress,  $f_s$  is the surface force,  $M$  is the bending moment, and  $A$  is the cross-sectional area of the beam element.

$= (y/a) \dots$  (53)

$$\frac{2E}{t} \int_A y^2 dA + S_{1111} t \dots = -M \tag{54}$$

$K \dots I = \int y^2 dA, \dots$

$$a = \frac{-M}{\frac{2EI}{t} + S_{1111} t} \tag{55}$$

$F \dots I = \frac{1}{12} t^3; \dots$  (55)

$$a = \frac{-M}{t(\frac{1}{6}Et + S_{1111})} \tag{56}$$

$\dots$

$$= \frac{-2My}{t^2(\frac{1}{6}Et + S_{1111})} \tag{57}$$

If  $\dots F a d d$





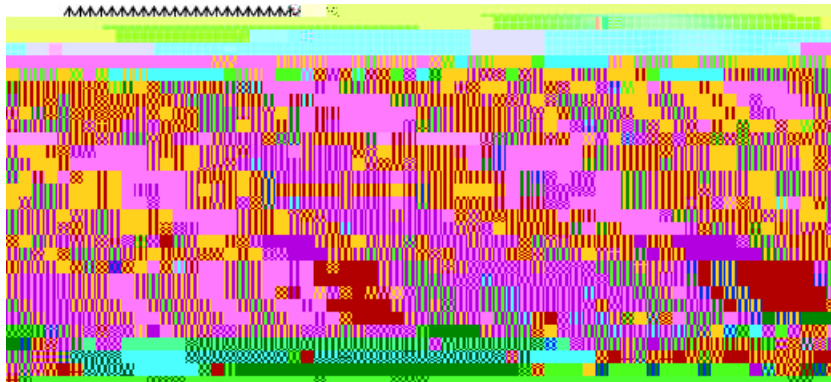
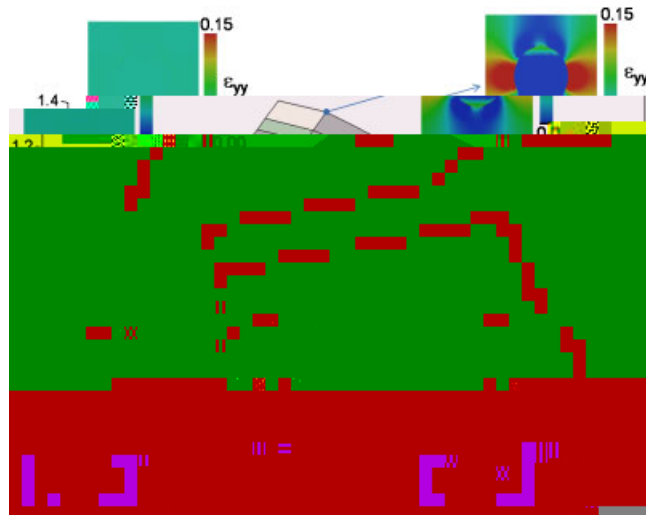


Figure 14. Generalized force field: (a) force field and (b) force field (with surface correction). Sub-figure (c) shows force field with surface correction.



• T... d... ab... d... c... b... c... a... a... b... a... c... a... c...  
 c... da... ar... I... a... f... f... d... f... a... d... f... ff...  
 d... f... a... a... d... f... ac... f... a... -c... c... c... c...

T... d... d... d... a... d... a... f... c... ca... a... f... f...  
 c... -d... c... a... a... f... a... a... a... d... a... -a... a... b... -a... a... a... f... f... c... ff... c...  
 a... a... ff... f... a... a... a... a... d... f... ac... ff... c... b... d... ff... f... a... a... b... a...  
 I... a... ca... XFEM... c... ca... c... a... c... a... d... d...  
 a... a... ca... F... c... a... da... ca... f... f... f... -d... c... a... a... f...  
 d... a... a... a... f... c... d... a... b... c... a... c... a... d... ca... SCB... ca... a...  
 F... a... a... a... a... c... b... a... f... a... a... d... c... d... c... a... d... ca... f...  
 a... ac... b... f... ac... d... c... a... d... f... ac... a... c... a... a... a... c... T...  
 c... f... f... ac... a... d... d... c... a... c... a... ca... c... f... c...  
 a... c... d... b... a... d... S... c... f... ac... a... c... a... a... ca... d... c...  
 b... d... a... d... d... c... a... a... c... a... a... c... b... c...  
 a... d... a... c... d... d... H... ff... c... b... d... f... ac... c...  
 a... d...

F... f... c... f... c... c... c... a... ff... c... f... a... c... a... d... f... ac... d... c...  
 d... f... a... [38-40], a... a... d... a... d... f... ac... ff... c... ac...  
 c... a... ca... f... a... a... a...

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